Two dimensional anelastic model deepconv-mars: Part 1, Governing equations

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Contents

1 Outline of the model

The 2D model system consists of the atmosphere and the grand surface. The model atmosphere is composed of ideal gas. The atmospheric constituent is assumed to CO_2 only and its condensation and sublimation are not considered. The values of soil density, thermal conductivity and specific heat are constant each other. The surface topography is not considered.

Atmospheric model The governing equation of model atmospheric dynamics is the 2D anelastic system (Ogura and Phillips, 1962).

Turbulent parametarization The subgrid turbulent mixingis evaluated by the formula of Klemp and Wilhelmson (1978). The value of turbulent diffusion coefficient for heat is equal to that for momentum.

Surface flux The surface momentum and heat fluxes are given by the bulk formulae, where the bulk coefficients depend on static stability and vertical wind shear (Louis, 1979). The bulk coefficient for heat transport has same value of that for momentum. The roughness length for the bulk coefficients is 1 cm (Sutton *et al.*, 1978).

Dust transport The spatial distribution of dust is calculated by advection diffusion equation which considers gravitational settling of dust. The representation of dust terminal velocity follows Conrath (1975). The radius of dust particle is constant value (0.4 μ m). The value of dust flux from the surface is constant.

Radiation The radiative transfer of CO_2 atmosphere is calculated by a Goody narrow band model which considers 15 μm band in infrared wavelength and 4.3, 2.7, 2.0 μm band in near infrared wavelength. The values of absorption line intensities and widths in each band are same as those of Houghton (1986).

The radiative transfer of dust is calculated by δ-Eddington approximation model which considers two bands in infrared wavelength region (5-11.6, 20-200 μm) and one band in solar wavelength region (0.1-5 μm). The values of band width and optical parameters of dust (extinction efficiency, single scattering albedo, asymmetry factor) in each band are same as those of Forget *et al.* (1999) except in 11.6-20 μm band which is not considered in our model.

Ground surface The ground temperature is calculated by 1D thermal conduction equation. The values of soil density, thermal conductivity and specific heat are same as those of Kieffer *et al.* (1977) standard model.

2 Atmospheric model

The governing equation of model atmospheric dynamics is the 2D anelastic system (Ogura and Phillips, 1962).

$$\frac{du}{dt} - fv = -c_p\Theta_0\frac{\partial \pi}{\partial x} + D(u), \tag{1}$$
$$\frac{dv}{dt} + fu = D(v), \tag{2}$$
$$\frac{dw}{dt} = -c_p\Theta_0\frac{\partial \pi}{\partial z} + g\frac{\theta}{\Theta_0} + D(w), \tag{3}$$
$$\frac{\partial(\rho_0 u)}{\partial x} + \frac{\partial(\rho_0 w)}{\partial z} = 0, \tag{4}$$
$$\frac{d\theta}{dt} + w\frac{\partial \Theta_0}{\partial z} = \frac{\Theta_0}{T_0}(Q_{rad} + Q_{dis}) + D(\theta + \Theta_0), \tag{5}$$
$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z}.$$

(1), (2), (3) are the horizontal and vertical component of equation of motion, respectively. (4) is the continuity equation and (5) is the thermodynamic equation. x, y, z, t are horizontal, vertical and time coordinate, respectively. u, v are horizontal and vertical velocity, and θ, π are potential temperature and nondimensional pressure function deviation from those of basic state, respectively. ρ_0, Θ_0, T_0 are density, potential temperature and temperature in basic state. g is gravitational acceleration whose value is equal to 3.72 msec^{-2}. Q_{rad} is radiative heating (cooling) rate per unit mass which is calculated by convergence of the radiative heat flux. Q_{dis} is heating rate per unit mass owing to dissipation of turbulent kinetic energy, which is given by turbulent parameterization.

$D(\cdot)$ term in equation (1)$\sim$(5)represents the turbulent diffusion owing to subgrid scale turbulent mixing.

$$D(\cdot) = \frac{\partial}{\partial x}\left[K\frac{\partial(\cdot)}{\partial x}\right] + \frac{1}{\rho_0}\frac{\partial}{\partial z}\left[\rho_0 K\frac{\partial(\cdot)}{\partial z}\right]. \tag{6}$$

K is turbulent diffusion coefficient which is calculated by (10) and (11).

The nondimensional pressure function and potential temperature are defined as follows.

$$\Pi \equiv \left(\frac{p}{P_{00}}\right)^{\kappa} = \Pi_0 + \pi, \quad \Pi_0 = \left(\frac{P_0}{P_{00}}\right)^{\kappa}$$
$$\Theta \equiv T\Pi^{-1} = \Theta_0 + \theta, \quad \Theta_0 = T_0\Pi_0^{-1}$$

where p and P_0 are pressure and basic state pressure, P_{00} is reference pressure ($= 7$ hPa), $\kappa = R/c_p$, c_p is specific heat of constant pressure per unit mass and R is atmospheric gas constant per unit mass. The basic state atmospheric structure is calculated by using the hydrostatic equation as follows.

$$\frac{dP_0}{dz} = -\rho_0 g, \tag{7}$$
$$P_0 = \rho_0 R T_0. \tag{8}$$

The deviation of nondimensional pressure function is calculated by using the following equation which is derived from (1)∼(4).

$$\begin{aligned} c_p\Theta_0\left[\rho_0\frac{\partial^2\pi}{\partial x^2} + \frac{\partial}{\partial z}\left(\rho_0\frac{\partial\pi}{\partial z}\right)\right] &= \frac{g}{\Theta_0}\frac{\partial(\rho_0\theta)}{\partial z} \\ &\quad -\frac{\partial}{\partial x}\left[\rho_0\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} - D(u)\right)\right] \\ &\quad -\frac{\partial}{\partial z}\left[\rho_0\left(u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} - D(w)\right)\right]. \end{aligned} \tag{9}$$

Boundary conditions

The model horizontal boundary is cyclic. The vertical wind velocity is set to be 0 at the surface and upper boundary.

Parameters

Table 1: Parameters of atmospheric model

Parameters	Standard Values	Note
f	0 sec^{-1}	
g	3.72 msec^{-2}	
P_{00}	7 hPa	
c_p	734.9 $\mathrm{Jkg}^{-1}\mathrm{K}^{-1}$	Value of CO_2
R	189.0 $\mathrm{Jkg}^{-1}\mathrm{K}^{-1}$	Value of CO_2

3 Turbulent parameterization

3.1 Subgrid turbulent mixing parameterization

The turbulent diffusion coefficient is evaluated by the formula of Klemp and Wilhelmson (1978), where the turbulent diffusion is proportional to square root of turbulent kinetic energy ε. The value of turbulent diffusion coefficient for heat is equal to that for momentum. The diagnostic equation of turbulent kinetic energy is as follows.

$$\frac{d\varepsilon}{dt} = BP + SP + D(\varepsilon) - \frac{C_\epsilon}{l}\varepsilon^{3/2}, \tag{10}$$

$$K = C_m\sqrt{\varepsilon}l. \tag{11}$$

$C_\epsilon = C_m = 0.2$. BP and SP are generation terms of the turbulent kinetic energy associated with buoyancy force and wind shear, respectively.

$$BP = -\frac{g}{\Theta_0}K\frac{\partial(\theta+\Theta_0)}{\partial z}, \tag{12}$$

$$SP = 2K\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \frac{2}{3}\varepsilon\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + K\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2, \tag{13}$$

where l is the mixing length which is the smaller value of either vertical grid interval or altitude.

$$l = \max(\Delta z, z).$$

The last term in left hand side of (10)represents the dissipation rate of turbulent kinetic energy. By using this term, Q_{dis} in equation (5)is given as follows.

$$Q_{dis} = \frac{C_\epsilon}{lc_p}\varepsilon^{3/2}. \tag{14}$$

3.2 Surface flux parameterization

The surface momentum and heat fluxes are given by the bulk formulae, where the bulk coefficients depend on static stability and vertical wind shear (Louis, 1979). The bulk coefficient for heat transport has same value of that for momentum.

$$F_u = -\rho_0 C_D |u_{z=z1}| u_{z=z_1}, \tag{15}$$

$$F_\theta = \rho_0 C_D |u_{z=z1}| (T_{sfc} - T_{z=z1}). \tag{16}$$

$u_{z=z1}, T_{z=z1}$ are the horizontal wind and temperature at the lowest level of the model atmosphere z_1. T_{sfc} is the surface temperature. The bulk coefficient C_D is calculated as follows.

$$C_D = \begin{cases} C_{Dn}\left(1 - \dfrac{a\mathrm{Ri}_B}{1 + c|\mathrm{Ri}_B|^{1/2}}\right) & for \quad \mathrm{Ri}_B < 0, \\ C_{Dn}\dfrac{1}{(1 + b\mathrm{Ri}_B)^2} & for \quad \mathrm{Ri}_B \geq 0, \end{cases} \tag{17}$$

where,

$$C_{Dn} = \left(\frac{k}{\ln(z_1/z_0)}\right)^2, \quad a = 2b = 9.4, \quad c = 0.74 \cdot ab\left(\frac{z_1}{z_0}\right)^{\frac{1}{2}}, \tag{18}$$

and k is the Karman constant ($= 0.35$), z_0 is the roughness length. Ri_B is the bulk Richardson number, which is given as follow.

$$\mathrm{Ri}_B \equiv \frac{gz_1(\Theta_{sfc} - \Theta_{z=z1})}{\overline{\Theta}_{z=z1} u_{z=z1}}, \tag{19}$$

where $\Theta_{z=z1}, \overline{\Theta}_{z=z1}$ and are the potential temperature and that of horizontal mean value at the lowest level of model atmosphere.

Parameters

Table 2: Parameters of surface flux parameterization

Parameters	Standard values	Note
k	0.35	
z_0	1 cm	Sutton *et al.*, (1978)

4 Dust transport

The spatial distribution of dust mass mixing ratio q is calculated by advection diffusion equation which considers gravitational settling of dust.

$$\frac{dq}{dt} + \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 W q) = D(q). \tag{20}$$

The dust terminal velocity W is calculated as follows (Conrath, 1975).

$$W = -\frac{4\rho_d g r^2}{18\eta}\left(1 + 2\frac{\lambda_r}{r}\frac{p_r}{P_0}\right). \tag{21}$$

ρ_d is the density of dust particle, r is the radius of dust particle, η is atmospheric viscosity, λ_r is the mean free path of atmospheric gas at p_r. The equation (21) is applied for each dust particle with different radius. However, the size distribution of dust is not considered here for simplicity and the radius of dust particle is supposed to be equal to the mode radius r_m of dust particle in equation (38)which is the size distribution function of dust.

It is supposed that the dust injection from the surface to atmosphere occurs when the surface stress $\tau_m \equiv |F_u|$ exceeds some threshold value. The value of dust flux from the surface is constant.

Parameters

Table 3: Parameters of dust transport model

Parameters	Standard Values	Note
ρ_d	3000 kgm^{-3}	Conrath (1975)
η	1.5×10^{-5} $kgm^{-1}sec^{-1}$	〃
p_r	25 hPa	〃
$\lambda_r(p_r)$	2.2×10^{-6}m	〃
r_m	0.4μm	Toon *et al.* (1977)
F_q	3.7×10^{-6} kgm^{-2}	White *et al.* (1997)
		$(\tau_m \geq \tau_{mc})$
τ_{mc}	0.01 Pa	

5 Radiation

Q_{rad} in equation (5) is given by convergence of net radiative heat flux which is calculated by using radiative transfer equation. We consider following radiation processes in this model; absorption of near infrared solar radiation (NIR), absorption and emission of infrared radiation associated with atmospheric CO_2, absorption and scattering of solar radiation, and absorption and emission of infrared radiation associated with dust.

Q_{rad} is represented as follows.

$$Q_{rad} = Q_{rad,IR} + Q_{rad,NIR} + Q_{rad,dust,SR} + Q_{rad,dust,IR}. \tag{22}$$

$Q_{rad,IR}$ and $Q_{rad,NIR}$ are the infrared and near infrared radiative heating rate associated with CO_2 $Q_{rad,dust,SR}$ and $Q_{rad,dust,IR}$ are the solar and infrared radiative heating rate associated with dust. The governing equations to calculate these heating rate are described in following sections.

5.1 Radiative transfer of atmospheric CO_2

Both infrared and near infrared radiative flux associated with CO_2 are calculated by Goody narrow band model (c.f., Goody and Young, 1989). In calculating infrared radiative flux, CO_2 15 μm band is only considered. The upward and downward infrared radiative flux $F_{IR}^{\uparrow}, F_{IR}^{\downarrow}$ and the infrared radiative heating rate per unit mass $Q_{rad,IR}$ are calculated as follows.

$$F^{\uparrow}_{IR}(z) = \sum_i \Delta\nu_i \left\{ \pi B_{\nu_i,T}(z=0)\mathcal{T}_i(0,z) + \int_0^z \pi B_{\nu_i,T}(z') \frac{d\mathcal{T}_i(z,z')}{dz'}\, dz' \right\}, \quad (23)$$

$$F^{\downarrow}_{IR}(z) = \sum_i \Delta\nu_i \left\{ \int_z^\infty \pi B_{\nu_i,T}(z') \frac{d\mathcal{T}_i(z,z')}{dz'}\, dz' \right\}, \quad (24)$$

$$Q_{rad,IR} = -\frac{1}{\rho_0 c_p} \frac{\partial}{\partial z} (F^{\uparrow}_{IR}(z) - F^{\downarrow}_{IR}(z)). \quad (25)$$

$\Delta\nu_i$ is the ith narrow band width and $B_{\nu_i,T}$ is the Plank function which is represented as follows.

$$B_{\nu_i,T} = \frac{2hc^2\nu_i^3}{e^{hc\nu_i/kT} - 1} = \frac{1.19 \times 10^{-8}\nu_i^3}{e^{1.4387\nu_i/T} - 1}, \quad (26)$$

where h is the Plank constant, c is speed of light, k is the Boltzmann constant, and T is temperature. $\mathcal{T}_i(z,z')$ is the transmission function averaged over $\Delta\nu_i$ around ν_i.

$$\mathcal{T}_i(z,z') = \exp(-W_i/\Delta\nu_i), \quad W_i = \frac{s_i u(z,z')}{\sqrt{1 + s_i u(z,z')/\alpha_i^*}},$$

$$u(z,z') = \int_z^{z'} 1.67\rho_0\, dz, \quad \alpha_i^* = \alpha_i \overline{p}/p_0, \quad \overline{p} = \int_z^{z'} P_0\, du/u.$$

s_i is line strength, α_i^* is square root of the product of line strength and line width and α_i is the reference value of α_i^*, u is effective path length, and p_0 is reference pressure (= 1013 hPa).

In calculating near infrared solar radiative flux, CO_2 4.3 μm, 2.7 μm, and 2.0 μm band are considered. The near infrared solar radiative flux $F^{\downarrow}_{NIR}$ and $Q_{rad,NIR}$ are calculated as follows.

$$F^{\downarrow}_{NIR}(z) = \sum_i \Delta\nu_i \left\{ S_{\nu_i} \mathcal{T}_i(\infty,z)\mu_0 \right\}, \quad (27)$$

$$Q_{rad,NIR} = \frac{1}{\rho_0} \frac{\partial F^{\downarrow}_{IR}(z)}{\partial z}, \quad (28)$$

where $\mu_0 = \cos\zeta$, ζ is the solar zenith angle, and S_{ν_i} is the solar radiative flux per unit wave length at the top of atmosphere which is represented as follows.

$$S_{\nu_i} = B_{\nu_i,T_{sol}} \left(\frac{F_s}{\sigma T_{sol}^4} \right), \quad (29)$$

$$F_s = I_0 \left(\frac{r_0}{r} \right)^2 \mu_0, \quad (30)$$

where T_{sol} is the surface temperature of the sun (= 5760 K), σ is the Stefan-Boltzmann constant (= 5.67×10^{-8} Wm^{-2}K^{-4}), I_0 is solar constant on the mean radius of Mars orbit (= 591 Wm^{-2}), r and r_0 is the radius of Mars orbit and its mean value, F_s is solar radiative flux at the top of atmosphere. F_s is depend on season, latitude and local time. Detail descriptions of F_s and $\cos\zeta$ are shown in Section 5.3.

The transmission function averaged over $\Delta\nu_i$ in near infrared wavelength region is similar to that in infrared wavelength region except for the effective path length u.

$$u(z, z') = \int_z^{z'} 1.67\rho_0 \, dz/\mu_0,$$

Parameters

The number of narrow band and its band width are similar to those of Savijärvi (1991a). The line strength and the square root of the product of line strength and line width are quoted from those at 220 K listed by Houghton (1986). These vaues are listed in Table 4 $\sim$ Table 7.

CO_2 15 μm band ranges from 500 cm^{-1} to 900 cm^{-1} and 4.3 μm band ranges from 2200 cm^{-1} to 2450 cm^{-1}, where $\Delta\nu_i$ is equal to 25 cm^{-1}. CO_2 2.7 μm band ranges from 3150 cm^{-1} to 4100 cm^{-1} and 4.0 μm band ranges from 4600 cm^{-1} to 5400 cm^{-1} , where $\Delta\nu$ is equal to 100 cm^{-1}.

5.2 Radiative transfer of dust

The solar and infrared radiative flux associated with dust are calculated by using the δ-Eddington approximation (c.f., Liou, 1980). The δ-Eddington approximation is well used in calculating radiative transfer with anisotropic

Table 4: Parameters of CO_2 15 μm band

ν_i(cm^{-1})	s_i	α_i	ν_i(cm^{-1})	s_i	α_i
512.5	1.952×10^{-2}	2.870×10^{-1}	712.5	1.232×10^{3}	8.387×10^{1}
537.5	2.785×10^{-1}	1.215×10^{0}	737.5	2.042×10^{2}	2.852×10^{1}
562.5	5.495×10^{-1}	2.404×10^{0}	762.5	7.278×10^{0}	6.239×10^{0}
587.5	5.331×10^{0}	1.958×10^{1}	787.5	1.337×10^{0}	2.765×10^{0}
612.5	5.196×10^{2}	5.804×10^{1}	812.5	3.974×10^{-1}	8.897×10^{-1}
637.5	7.778×10^{3}	2.084×10^{2}	837.5	1.280×10^{-2}	3.198×10^{-1}
662.5	8.746×10^{4}	7.594×10^{2}	862.5	2.501×10^{-3}	1.506×10^{-1}
687.5	2.600×10^{4}	2.635×10^{2}	887.5	3.937×10^{-3}	1.446×10^{-1}

Table 5: Parameters of CO_2 4.3 μm band

ν_i(cm^{-1})	s_i	α_i	ν_i(cm^{-1})	s_i	α_i
2212.5	9.504×10^{-1}	2.866×10^{0}	2337.5	5.587×10^{5}	1.206×10^{3}
2237.5	2.217×10^{2}	3.000×10^{1}	2362.5	6.819×10^{5}	1.182×10^{3}
2262.5	4.566×10^{3}	1.134×10^{2}	2387.5	1.256×10^{4}	8.873×10^{1}
2287.5	7.965×10^{3}	2.011×10^{2}	2412.5	7.065×10^{-1}	3.404×10^{-1}
2312.5	1.055×10^{5}	5.880×10^{2}	2437.5	8.522×10^{-2}	4.236×10^{-1}

Table 6: Parameters of CO_2 2.7 μm band

ν_i(cm^{-1})	s_i	α_i	ν_i(cm^{-1})	s_i	α_i
3150	1.324×10^{-1}	9.836×10^{-1}	3650	1.543×10^{4}	3.245×10^{2}
3250	7.731×10^{-2}	4.900×10^{-1}	3750	1.649×10^{4}	2.722×10^{2}
3350	1.232×10^{0}	2.952×10^{0}	3850	1.180×10^{-1}	9.535×10^{-1}
3450	5.159×10^{0}	7.639×10^{0}	3950	1.464×10^{-2}	2.601×10^{-1}
3550	4.299×10^{3}	1.914×10^{2}	4050	1.251×10^{-2}	2.021×10^{-1}

Table 7: Parameters of CO_2 2.0 μm band

ν_i(cm^{-1})	s_i	α_i	ν_i(cm^{-1})	s_i	α_i
4650	2.185×10^{-1}	1.916×10^{0}	5050	8.778×10^{1}	2.012×10^{1}
4750	2.040×10^{0}	6.475×10^{0}	5150	8.346×10^{1}	1.804×10^{1}
4850	1.197×10^{2}	3.112×10^{1}	5250	8.518×10^{-2}	8.474×10^{-1}
4950	4.829×10^{2}	5.759×10^{1}	5350	4.951×10^{-1}	1.597×10^{0}

scattering. The asymmetry factor of dust for solar and infrared radiation are between 0 and 1 which means forward scattering occurs.

The upward and downward diffuse solar radiative flux per unit wave length associated with dust $F^{\uparrow}_{dif,\nu_i}$, $F^{\downarrow}_{dif,\nu_i}$ are obtained as solutions of following equations.

$$\frac{dF^{\uparrow}_{dif,\nu_i}}{d\tau^*_{\nu_i}} = \gamma_{1,\nu_i} F^{\uparrow}_{dif,\nu_i} - \gamma_{2,\nu_i} F^{\downarrow}_{dif,\nu_i} - \gamma_{3,\nu_i} \tilde{\omega}^*_{\nu_i} S_{\nu_i} e^{-\tau^*_{\nu_i}/\mu_0}, \quad (31)$$

$$\frac{dF^{\downarrow}_{dif,\nu_i}}{d\tau^*_{\nu_i}} = \gamma_{2,\nu_i} F^{\uparrow}_{dif,\nu_i} - \gamma_{1,\nu_i} F^{\downarrow}_{dif,\nu_i} + (1-\gamma_{3,\nu_i}) \tilde{\omega}^*_{\nu_i} S_{\nu_i} e^{-\tau^*_{\nu_i}/\mu_0}. \quad (32)$$

The boundary condition of (31) and (32)are that $F^{\downarrow}_{dif,\nu_i} = 0$ at the top of atmosphere and $F^{\uparrow}_{dif,\nu_i} = F^{\downarrow}_{dif,\nu} \times A$ at the surface, where A is the surface albedo. $\gamma_{1,\nu_i}, \gamma_{2,\nu_i}, \gamma_{3,\nu_i}$ are expressed as follows.

$$\gamma_{1,\nu_i} = \frac{1}{4}[7-(4+3g^*_{\nu_i})\tilde{\omega}^*_{\nu_i}], \quad \gamma_{2,\nu_i} = -\frac{1}{4}[1-(4-3g^*_{\nu_i})\tilde{\omega}^*_{\nu_i}], \quad \gamma_{3,\nu_i} = \frac{1}{4}(2-3g^*_{\nu_i}\mu_0),$$

where $\tau^*_{\nu_i}, \tilde{\omega}^*_{\nu_i}, g^*_{\nu_i}$ are optical depth, single scattering albedo and asymmetry factor scaled by δ-Eddington approximation, which are given as follows.

$$\tau^*_{\nu_i} = (1-\tilde{\omega}_{\nu_i} g^2_{\nu_i})\tau_{\nu_i}, \quad \tilde{\omega}^*_{\nu_i} = \frac{(1-g^2_{\nu_i})\tilde{\omega}_{\nu_i}}{1-\tilde{\omega}_{\nu_i} g^2_{\nu_i}}, \quad g^*_{\nu_i} = \frac{g_{\nu_i}}{1+g_{\nu_i}},$$

where $\tau_{\nu_i}, \tilde{\omega}_{\nu_i}, g_{\nu_i}$ are optical depth, single scattering albedo and asymmetry factor, respectively.

The upward and downward infrared radiative flux per unit wave length associated with dust are obtained as solutions of similar equations used for calculation of diffuse solar flux ((31), (32)) except for the last term in right hand side of each equation.

$$\frac{dF^{\uparrow}_{IR,\nu_i}}{d\tau^*_{\nu_i}} = \gamma_{1,\nu_i} F^{\uparrow}_{IR,\nu_i} - \gamma_{2,\nu_i} F^{\downarrow}_{IR,\nu_i} - 2\pi(1-\tilde{\omega}^*_{\nu_i}) B_{\nu_i,T}(\tau^*_{\nu_i}), \quad (33)$$

$$\frac{dF^{\downarrow}_{IR,\nu_i}}{d\tau^*_{\nu_i}} = \gamma_{2,\nu_i} F^{\uparrow}_{IR,\nu_i} - \gamma_{1,\nu_i} F^{\downarrow}_{IR,\nu_i} + 2\pi(1-\tilde{\omega}^*_{\nu_i}) B_{\nu_i,T}(\tau^*_{\nu_i}). \quad (34)$$

The boundary condition of (33)and (34)is that $F^{\downarrow}_{IR,\nu_i} = 0$ at the top of atmosphere and $F^{\uparrow}_{IR,\nu_i}$ is equal to $\pi B_{\nu,T_{sfc}}$ at the surface. The Plank function $B_{\nu_i,T}$ in (33)and (34) is averaged over the band width.

$$B_{\nu_i,T} = \frac{1}{\nu_2 - \nu_1} \int_{\nu_1}^{\nu_2} B_{\nu,T}\, d\nu.$$

ν_1, ν_2 are the lower and upper wave length of the band.

The radiative heating rate associated with dust is calculated as follows.

$$Q_{rad,dust,SR} = -\frac{1}{\rho_a c_p}\frac{d}{dz}\left[\sum_{\nu_i} \Delta\nu_i \left(F^{\uparrow}_{dif,\nu_i} - F^{\downarrow}_{dif,\nu_i} - F^{\downarrow}_{dir,\nu_i}\right)\right], \quad (35)$$

$$Q_{rad,dust,IR} = -\frac{1}{\rho_a c_p}\frac{d}{dz}\left[\sum_{\nu_i} \Delta\nu_i \left(F^{\uparrow}_{IR,\nu_i} - F^{\downarrow}_{IR,\nu_i}\right)\right]. \quad (36)$$

$F^{\downarrow}_{dir,\nu_i}$ is the direct solar radiative flux per unit wave length,

$$F^{\downarrow}_{dir,\nu_i} = \mu_0 S_{\nu_i} e^{-\tau_{\nu_i}/\mu_0} \quad (37)$$

The dust opacity is calculated by using the mass mixing ratio and effective radius of dust. In this model, we suppose that the size distribution of dust particle is the modified gamma distribution (Toon *et al.*, 1977).

$$\frac{dn(r)}{dr} = n_0 r^{\alpha} \exp\left[-\left(\frac{\alpha}{\gamma}\right)\left(\frac{r}{r_m}\right)^{\gamma}\right]. \quad (38)$$

Dust opacity

The monoclomatic optical depth τ_ν is represented by using the extinction coefficient per unit volume $\beta_{e,\nu}$ as follows.

$$\tau_\nu(z) = -\int_{z_t}^{z} \beta_{e,\nu}(r)\, dz \quad (39)$$

where z_t is altitude at the top of atmosphere. $\beta_{\nu,e}$ is given as follows.

$$\beta_{e,\nu} = \int_0^{\infty} \sigma_{e,\nu}(r) \frac{dn(r)}{dr}\, dr \quad (40)$$

where $\sigma_{e,\nu}$ is the extinction cross section, $dn(r)/dr$ is the size distribution of scattering particle (cf. Liou, 1980; Shibata, 1999). By using extinction coefficient per unit mass k_e, (40) is rewritten as follows.

$$\rho_a q_s k_{e,\nu} = \int_0^\infty \sigma_{e,\nu}(r) \frac{dn(r)}{dr}\, dr \tag{41}$$

where ρ_a is atmospheric density, and q_s is mass mixing ratio of scattering particle. Similarly, the scattering and absorption coefficient per unit volume are represented by using the scattering cross section $\sigma_{s,\nu}$ and the absorption cross section $\sigma_{a,\nu}$ as follows.

$$\beta_{s,\nu} = \int_0^\infty \sigma_{s,\nu}(r) \frac{dn(r)}{dr}\, dr, \tag{42}$$

$$\beta_{a,\nu} = \int_0^\infty \sigma_{a,\nu}(r) \frac{dn(r)}{dr}\, dr, \tag{43}$$

and the single scattering albedo $\tilde{\omega}_\nu$ is given as follows.

$$\tilde{\omega}_\nu = \frac{\beta_{s,\nu}}{\beta_{a,\nu}} \tag{44}$$

The extinction efficiency $Q_{e,\nu}$ is defined as the ration of extinction cross section to geometric cross section.

$$Q_{e,\nu} = \frac{\sigma_{e,\nu}}{\pi r^2}, \tag{45}$$

Similarly, the scattering efficiency $Q_{s,\nu}$ and absorption efficiency $Q_{a,\nu}$ is defined as follows.

$$Q_{s,\nu} = \frac{\sigma_{s,\nu}}{\pi r^2}, \tag{46}$$

$$Q_{a,\nu} = \frac{\sigma_{a,\nu}}{\pi r^2}. \tag{47}$$

In this model, the dust opacity is derived from the mass mixing ratio of atmospheric dust. Given parameters are the cross section weighted mean extinction efficiency $\overline{Q}_{e,\nu}$, the single scattering albedo $\tilde{\omega}_\nu$, the size distribution function of dust $dn(r)/dr$, the mode radius r_m, the effective (or, cross section weighted mean) radius r_{eff}, and the density of dust particle ρ_d. $\overline{Q}_{e,\nu}$ and r_{eff} are defined as follows, respectively.

$$\overline{Q}_{e,\nu} \equiv \frac{\int_0^\infty Q_{e,\nu}\pi r^2 \frac{dn(r)}{dr}\,dr}{\int_0^\infty \pi r^2 \frac{dn(r)}{dr}\,dr}, \quad (48)$$

$$r_{eff} \equiv \frac{\int_0^\infty r^3 \frac{dn(r)}{dr}\,dr}{\int_0^\infty r^2 \frac{dn(r)}{dr}\,dr}, \quad (49)$$

Supposing that the shape of scattering particle is sphere, the extinction coefficient per unit mass is given as follows.

$$\begin{aligned} \beta_{e,\nu} &= \overline{Q}_{e,\nu}\int_0^\infty \pi r^2 \frac{dn(r)}{dr}\,dr, \\ &= \frac{\overline{Q}_{e,\nu}}{r_{eff}}\int_0^\infty \pi r^3 \frac{dn(r)}{dr}\,dr, \\ &= \frac{\overline{Q}_{e,\nu}}{r_{eff}}\frac{3\rho_a q_s}{4\pi\rho_d}, \end{aligned} \quad (50)$$

where ρ_a is the atmospheric density. Therefore, the optical depth can be represented by using the mass mixing ratio as follows.

$$\tau_\nu = -\int_{z_t}^{z} \frac{\overline{Q}_{e,\nu}}{r_{eff}}\frac{3\rho_a q_s}{4\pi\rho_s}\,dz, \quad (51)$$

Parameters

The values of band width and optical parameters of dust (extinction efficiency, single scattering albedo, asymmetry factor) considered in this model are following to those of Forget *et al.* (1999) except for 11.6–20 μm band of dust. The overlap between visible band of dust and CO_2 near infrared band is omitted.

The 5-11.6 μm infrared dust opacity $\tau_{5-1.6\mu m}$ is obtained by dividing the visible dust opacity by the visible to infrared opacity ratio $\tau_{0.67\mu m}/\tau_{9\mu m}$, which is set to be 2. (Forget, 1998). The 20-200 μm infrared dust opacity is calculated by using $\tau_{5-11.6\mu m}$ and the value of $Q_{e,\nu_i}/Q_{e,0.67\mu m}$ shown in Table 8.

Table 8: Band width and optical parameters of dust

Band(μm)	Band(cm^{-2})	$Q_{e,\nu_i}/Q_{e,0.67\mu m}$	$\tilde{\omega}_{\nu_i}$	g_{ν_i}
0.1–5 μm	2000–10^5	1.0	0.920	0.55
5–11.6 μm	870–2000	0.253	0.470	0.528
20–200 μm	50–500	0.166	0.370	0.362

Table 9: Other parameters

Parameters	Standard values	Note
$Q_{e,0.67\mu m}$	3.04	Ockert-Bell, *et al.* (1997)
$\tau_{0.67\mu m}/\tau_{9\mu m}$	2	Forget (1998)
r_{eff}	2.5 μm	Pollack *et al.* (1979)
r_m	0.4 μm	Pollack *et al.* (1979)

5.3 Solar flux and zenith angle

The solar flux at the top of atmosphere F_s is depend on season, latitude and local time. In this section, we show F_s as a function of local time at a specified season and latitude.

Suppose that I_0 (Wm^{-2}) is solar constant on the mean orbital radius of Planet, r and r_0 is the radius of orbit and its mean value, ζ is solar zenith angle, ϕ is latitude, δ is the solar inclination, h is the hour angle ($= 2\pi t/T - \pi$, T is length of day). F_s is represented by using these variables as follows.

$$F_s = I_0 \left(\frac{r_0}{r}\right)^2 \cos\zeta, \tag{52}$$

$$\cos\zeta = \sin\phi\sin\delta + \cos\phi\cos\delta\cos h, \tag{53}$$

(c.f., Ogura, 1999). r and δ are given as follows.

$$r = \frac{a(1-e^2)}{1+e\cos\omega},$$

$$\sin\delta = \sin\alpha\sin(\omega - \omega_0)$$

where θ is the longitude relative to the perihelion, a is the semimajor axis of orbit, e is the eccentricity, α is the declination, ω; is the true anomaly, and ω_0 is the longitude of vernal equinox relative to the perihelion. By introducing the areocentric longitude of the sun $L_s \equiv \omega - \omega_0$, F_s is rewritten as follows.

$$F_s = I_0 \left(\frac{1 + e\cos(L_s + \omega_0)}{1 + e^2} \right)^2 \left[\sin\phi \sin\alpha \sin L_s + \cos h \cos\phi \sqrt{1 - \sin^2\alpha \sin^2 L_s} \right] \quad (54)$$

Parameters

Table 10: Parameters for solar flux and zenith angle

Parameters	Standard values	Note
ϕ	20°N	Pollack *et al.* (1979)
L_s	100 °	〃
e	0.093	
α	25.2°	
ω_0	110°	Carr (1996), Fig. 1
I_0	591 $\mathrm{Wm^{-2}}$	

6 Ground surface

The grand temperature is calculated by the 1D thermal conduction equation.

$$\rho_g c_{p,g} \frac{\partial T_g}{\partial t} = k_g \frac{\partial^2 T_g}{\partial z^2}. \tag{55}$$

where T_g is the grand temperature (K), ρ_g is the soil density (kgm^{-3}), $c_{p,g}$ is the specific heat of soil (Jkg^{-1}K^{-1}), and k_g is the thermal conductivity (Wm^{-1}K^{-1}). The surface temperature T_{sfc} is given by $T_{sfc} = T_g|_{z=0}$.

The boundary condition at the surface is given as follows.

$$-k \left.\frac{\partial T}{\partial z}\right|_{z=0} = -F_{SR}(1-A) + F_{IR,net} + H, \tag{56}$$

where F_{SR} is the solar radiative flux at the surface (the sign of downward flux is positive), A is the surface albedo, $F_{IR,net}$ is the net infrared radiative flux emitted from the surface and H is the sensible heat flux (the sign of upward flux is positive). The lower boundary of the grand surface is given as a insulation boundary.

Parameters

The values of soil density, thermal conductivity and specific heat are same as those of standard model of Kieffer *et al.* (1977).

Table 11: Parameters of ground surface model

Parameters	Standard values	Note
A	0.25	Kieffer *et al.* (1977)
ρ_g	1650 kgm^{-3}	〃
$c_{p,g}$	588 JK^{-1}kg^{-1}	〃
k_g	7.63×10^{-2}JK^{-1}m^{-1}sec^{-1}	〃

By using these values, the thermal inertia $I \equiv \sqrt{\rho_g c_{p,g} k_g}$ is 272 Wm^{-2}sec$^{1/2}$K^{-1} and the diurnal skin depth of $\delta_d \equiv \sqrt{k_g t_d/(\rho_g c_{p,g})}$ is about 8.2 cm.

References

Carr, M. H. 1996: *Water on Mars*, Oxfird Univ. Press, 229pp.

Conrath, B. J., 1975: Thermal structure of the Martian atmosphere during the dissipation of the dust storm of 1971, *Icarus*, **24**, 36–46.

Forget, F., 1998: Improved optical properties of the Martian atmospheric dust for radiative transfer calculations in the infrared, *Geopys. Res. Lett.*, **25**, 1105–1108.

Forget, F., F. Hourdin, R. Fournier, C. Hourdin, O. Talagrand, M. Collins, S. R. Lewis, P. L. Read, and J. P. Huot, 1999: Improved general circulation models of the Martian atmosphere from the surface to above 80 km, *J. Geophys. Res.*, **104**, 24155–24175.

Goody, R. M., and Y. L. Yung, 1989: *Atmospheric Radiation (Theoritcal basis)*, 2'nd ed., Oxford Univ. Press, 519pp.

Houghton, J. T., 1986: *The Physiscs of Atmospheres*, 2'nd ed., Cambridge Univ. Press, 271pp.

Kieffer, H. H., T. Z. Martin, A. R. Peterfreund, B. M. Jakosky, E. D. Miner, and F. D. Palluconi, 1977: Thermal and albedo mapping of Mars during the Viking primary mission, *J. Geophys. Res.*, **82**, 4249–4291.

Klemp, J. B., and R. B. Wilhelmson, 1978: The simulation of three-dimensional convective storm dynamics. *J. Atmos. Sci.*, **35**, 1070–1096.

Liou, K-N. 1980: *An Introduction to Atmosphric Radiation*, Academic Press, 392pp.

Louis, J. 1979: A parametric model of vertical eddy fluxes in the atmosphere. *Bound.-Layer Meteor.*, **17**, 187–202.

Nakajima, K. 1994: Direct numerical experiments on the large-scale organizations of cumulus convection, Ph.D thesis, Department of Earth and Planetary Science, Graduate School of Science, University of Tokyo, Tokyo, Japan(in Japanese).

Ockert-Bell, M. E., J. F. Bell III, J. B. Pollack, C. P. McKay, and F. Forget, 1997: Absorption and scattering properties of the Martian dust in the solar wavelengths, *J. Geophys. Res.*, **102**, 9039–9050.

Ogura, Y., 1999: Ippan Kishougaku, 2nd ed., Univ. Tokyo Press (in Japanese).

Ogura, Y., and N. A. Phillips, 1962: Scale analysis of deep and shallow convection in the atmosphere, *J. Atmos. Sci.*, **19**, 173–179.

Odaka M., K. Nakajima, M. Ishiwatari, and Y.-Y. Hayashi, 2001: A numerical simulation of thermal convection in the Martian lower atmosphere with a two-dimensional anelastic model, Nagare multimedia, 2001, http://www.nagare.or.jp/mm/2001/odaka/

Pollack, J. B., D. S. Colburn, F. M. Flasar, R. Kahn, C. E. Carlston, and D. Pidek, 1979: Properties and effects of dust particles suspended in the Martian atmosphere, *J. Geophys. Res.*, **84**, 2929–2945.

Savijärvi, H. 1991a: Radiative fluxes on dustfree Mars, *Contr. Atmosph. Phys.*, **64**, 103–112.

Shibata, K., 1999: Hikarino Kishougaku, Asakura (in Japanese).

Sutton, J. L., C. B. Leovy, and J. E. Tillman, 1978: Diurnal variations of the Martian surface layer meteorological parameters during the first 45 sols at two Viking lander sites, *J. Atmos. Sci.*, **35**, 2346–2355.

Toon, O. B., J. B. Pollack, and C. Sagan, 1977: Physical properties of the particles composing the Martian dust storm of 1971–1972, *Icarus*, **30**, 663–693.

White, B. R., B. M. Lacchia, R. Greeley, and R. N. Leach, 1997: Aeolian behavior of dust in a simulated Martian environment, *J. Geophys. Res.*, **102**, 25629–25640.