Waves and their role in the general circulation of the atmosphere

- 1 Nonrotating stratified flow: internal gravity waves and vertical momentum transport
- 2 Quasigeostrophic flow: Waves, instability, and momentum transport
- 3 The circulation of the stratosphere and mesosphere
- 4 Stirring and mixing in the stratosphere: Transport time scales and the distribution of trace gases
- 5 Eddies and tropospheric climate

FDEPS 2010

Alan Plumb, MIT

Nov 2010

Lecture 1:

Nonrotating stratified flow: internal gravity waves and vertical momentum transport

(i) 2D nonrotating, stratified flow

(ii) Internal gravity waves

(iii) momentum transport

(iv) internal gravity wave breaking

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(i) 2D nonrotating, stratified flow

Log-pressure coordinates for hydrostatic, compressible, flowx

log-pressure coordinates, pseudoheight $\boxed{z(p) = -H \ln p}$

hydrostatic balance (appropriate for large scale, low-frequency waves)
(z_g is *geometric* height; ρ_g is density in geometric coordinates)

$$\partial p / \partial z_g = -g \rho_g$$

constant $H = RT_* / g$, where T_* is constant reference temperature

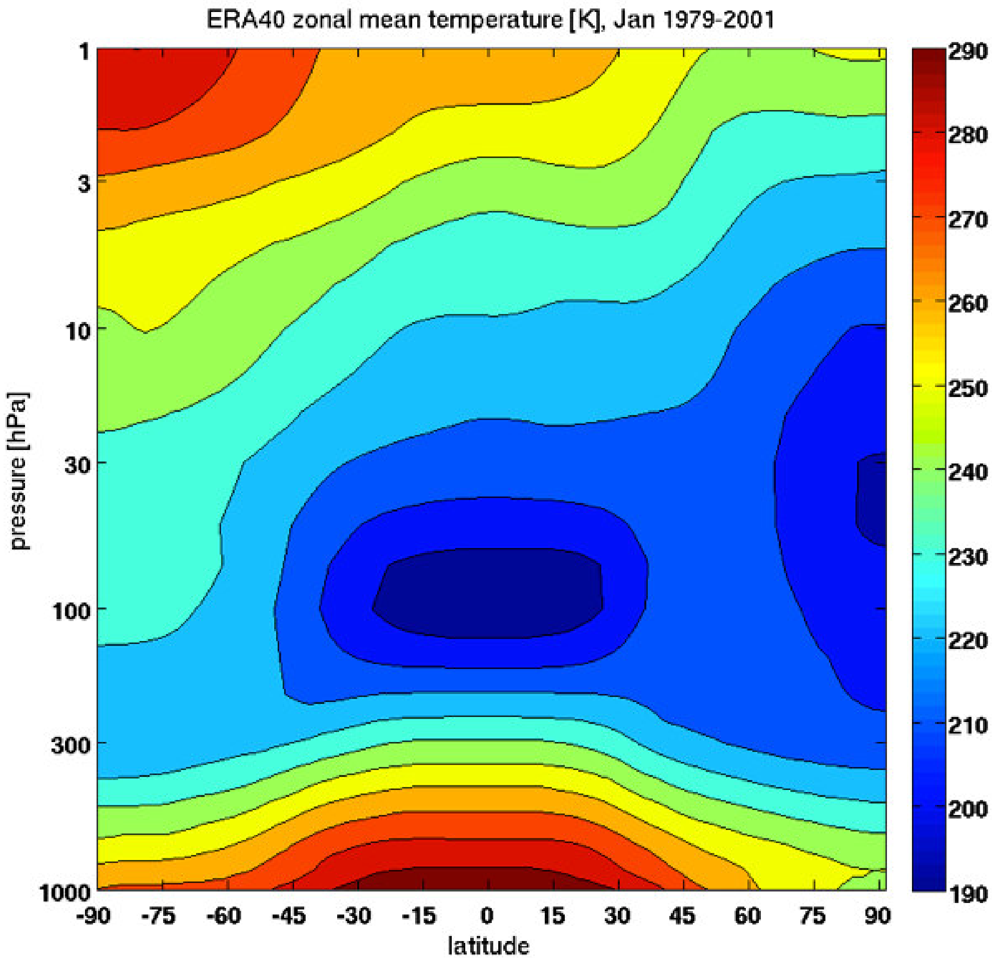
$$\rightarrow \quad dz = -H \frac{dp}{p} = gH \frac{\rho}{p} dz_g = \frac{T_*}{T} dz_g \qquad \left(\left| \frac{T}{T_*} - 1 \right| < 0.2 \right)$$

potential temperature $\boxed{\theta = T(p_*/p)^{\kappa}}$

where p_*= constant (1000hPa) and $\kappa = R/c_p = 2/7$
(specific entropy = $c_p \ln\theta$ + constant)

$\rightarrow \quad c_p T = \Pi(p)\theta \quad$ where $\Pi(p) = c_p (p/p_*)^{\kappa}$ is the Exner function

January climatology of T



190K < T < 300K

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$\rightarrow \quad c_p T = \Pi(p) \theta \qquad$ where $\Pi(p) = c_p (p / p_*)^{\kappa}$ is the Exner function

Two-dimensional hydrostatic, compressible, nonrotating flow

(1) momentum
pressure gradient force per unit mass

$$-\frac{1}{\rho_g}\left(\frac{\partial p}{\partial x}\right)_{z_g} = \frac{1}{\rho_g}\left(\frac{\partial p}{\partial z_g}\right)\left(\frac{\partial z_g}{\partial x}\right)_p = -\frac{\partial \phi}{\partial x} \quad ; \quad \phi = g z_g$$

$$\rightarrow \boxed{\frac{du}{dt} = -\frac{\partial \phi}{\partial x} + F}$$; F is other (*e.g.* frictional) force per unit mass)

(2) mass continuity
mass element is $\rho_g \, dx \, dy \, dz_g = \frac{p(z)}{gH} \, dx \, dy \, dz$
so log-p coordinate density is

$$\rho = \frac{p}{gH} \quad \rightarrow \quad \rho \text{ constant at constant } p$$

$$p(z) = p_0 \exp\left(-\frac{z}{H}\right) \rightarrow \rho(z) = \rho_0 \exp\left(-\frac{z}{H}\right) \;, \quad \rho_0 = \frac{p_0}{gH}$$

→ mass per unit area between coordinate surfaces z, $z + dz$ constant,
so mass flux is nondivergent:

$$\boxed{\nabla \cdot (\rho \mathbf{u}) = 0}$$

(3) entropy budget

$$\rho c_p \frac{dT}{dt} - \frac{dp}{dt} = J \quad \rightarrow \quad \boxed{\frac{d\theta}{dt} = (\rho\Pi)^{-1} J}$$

(J is heating rate per unit volume)

(4) hydrostatic balance

$$\frac{\partial z_g}{\partial p} = -\frac{1}{g\rho_g}$$

$$\frac{\partial \phi}{\partial z} = \frac{g\,\partial z_g}{-Hp^{-1}\partial p} = \frac{gp}{H}\frac{1}{g\rho_g} = \frac{R}{H}T \qquad \text{(ideal gas law)}$$

$$\rightarrow \boxed{\frac{\partial \phi}{\partial z} = \frac{\kappa\Pi}{H}\theta}$$

Two-dimensional hydrostatic, compressible, nonrotating flow

Full set of equations

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{\partial \phi}{\partial x} + F$$

$$\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + w\frac{\partial \theta}{\partial z} = (\rho\Pi)^{-1}J$$

$$\frac{\partial \phi}{\partial z} - \frac{\kappa}{H}\Pi\,\theta = 0$$

(ii) Internal gravity waves

2D internal gravity waves in a compressible fluid (simplest case)

inviscid, adiabatic ($F = 0 = J$)
motionless basic state

$$\theta = \theta_0(z)$$

$$\phi_0(z) = \kappa H^{-1} \int_0^z \theta_0(z')\Pi(z')\, dz'$$

small amplitude perturbations $\varepsilon << 1$
[neglect terms $O(\varepsilon^2)$]

$$\frac{\partial u'}{\partial t} + \frac{\partial \phi'}{\partial x} = 0$$

$$\frac{\partial u'}{\partial x} + \frac{1}{\rho}\frac{\partial(\rho w')}{\partial z} = 0$$

$$\frac{\partial \theta'}{\partial t} + w'\frac{d\theta_0}{dz} = 0$$

$$\frac{\partial \phi'}{\partial z} - \frac{\kappa}{H}\Pi\, \theta' = 0$$

All coefficients are functions of z, look for solutions

$$\begin{pmatrix} u' \\ w' \\ \phi' \\ \theta' \end{pmatrix} = \mathrm{Re} \begin{pmatrix} U(z) \\ W(z) \\ \Phi(z) \\ \Theta(z) \end{pmatrix} \exp[i(kx + ly - \omega t)]$$

$$\begin{aligned} \frac{\partial u'}{\partial t} + \frac{\partial \phi'}{\partial x} &= 0 \\ \frac{\partial u'}{\partial x} + \frac{1}{\rho}\frac{\partial(\rho w')}{\partial z} &= 0 \\ \frac{\partial \theta'}{\partial t} + w'\frac{d\theta_0}{dz} &= 0 \\ \frac{\partial \phi'}{\partial z} - \frac{\kappa}{H}\Pi\,\theta' &= 0 \end{aligned}$$

$\longrightarrow$

$$\begin{aligned} -i\omega U + ik\Phi &= 0 \\ ikU + \frac{1}{\rho}\frac{d}{dz}(\rho W) &= 0 \\ -i\omega\Theta + W\frac{d\theta_0}{dz} &= 0 \\ \frac{d\Phi}{dz} - \frac{\kappa}{H}\Pi\Theta &= 0 \end{aligned}$$

Reduce to single equation for Φ:

$$e^{z/H}\frac{d}{dz}\left(\frac{\omega^2}{N^2}e^{-z/H}\frac{d\Phi}{dz}\right) + (k^2 + l^2)\Phi = 0$$

where

$$N^2(z) = \frac{\kappa}{H}\Pi\frac{d\theta_0}{dz} = \frac{g}{T_*}\left(\frac{dT_0}{dz} + \frac{\kappa}{H}T_0\right)$$

→ square of *buoyancy frequency*

Solution for constant N^2:

$$\phi' = \mathrm{Re}\,\Phi_0 \exp(\frac{z}{2H}) \exp[i(kx + mz - \omega t)\}$$

where

$$m = \pm\sqrt{\frac{N^2k^2}{\omega^2} - \frac{1}{4H^2}}$$

or

$$\omega = \pm N\sqrt{\frac{k^2}{m^2 + 1/4H^2}}$$

Note that if m real:
(i) wave propagates in vertical
and
(ii) grows with height as
$e^{z/2H} \sim \rho^{-1/2}$

Assume $m^2 \gg 1/4H^2 \rightarrow 2\pi/m \ll 4\pi H \simeq 100\text{km}$
— good assumption for important atmospheric waves

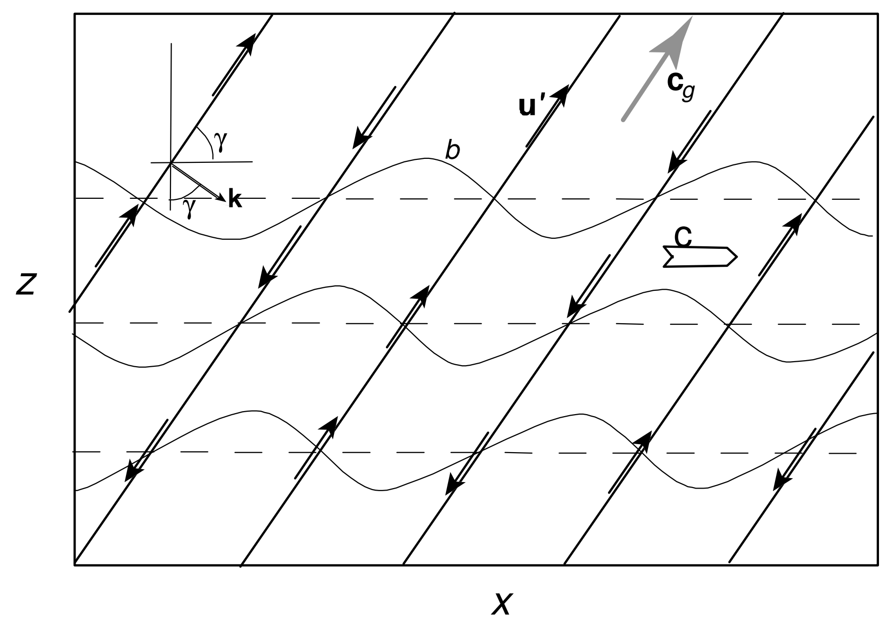
$$\omega = \pm N\frac{k}{m} = \pm N\tan\gamma$$

$(\gamma = \tan^{-1}k/m)$; nonhydrostatic case: $\omega = \pm N\sin\gamma$
group velocity:

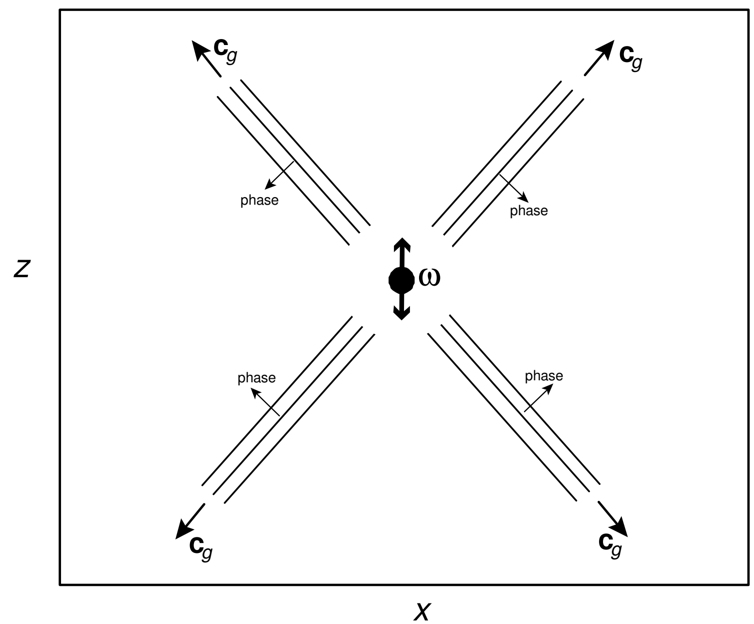
(hydrostatic approximation valid for $\omega \ll N$)

$$\mathbf{c}_g = \left(\frac{\partial\omega}{\partial k}, \frac{\partial\omega}{\partial m}\right) =. \pm\frac{N}{m}\left(1, -\frac{k}{m}\right)$$

(i) $\rightarrow$ $\mathbf{c}_g \cdot \mathbf{k} = 0$: group propagation is *along* phase lines, at angles $\pm\gamma$
(ii) continuity eq.$\rightarrow$ $\mathbf{k} \cdot \mathbf{u}' = 0$ – fluid motions are along phase lines
(iii) vertical components of group and phase velocities have *opposite* signs.



From localized source of frequency ω, waves form rays at angles $\gamma = \sin^{-1}(\omega/N)$ to horizontal, with phase propagation *across* rays:



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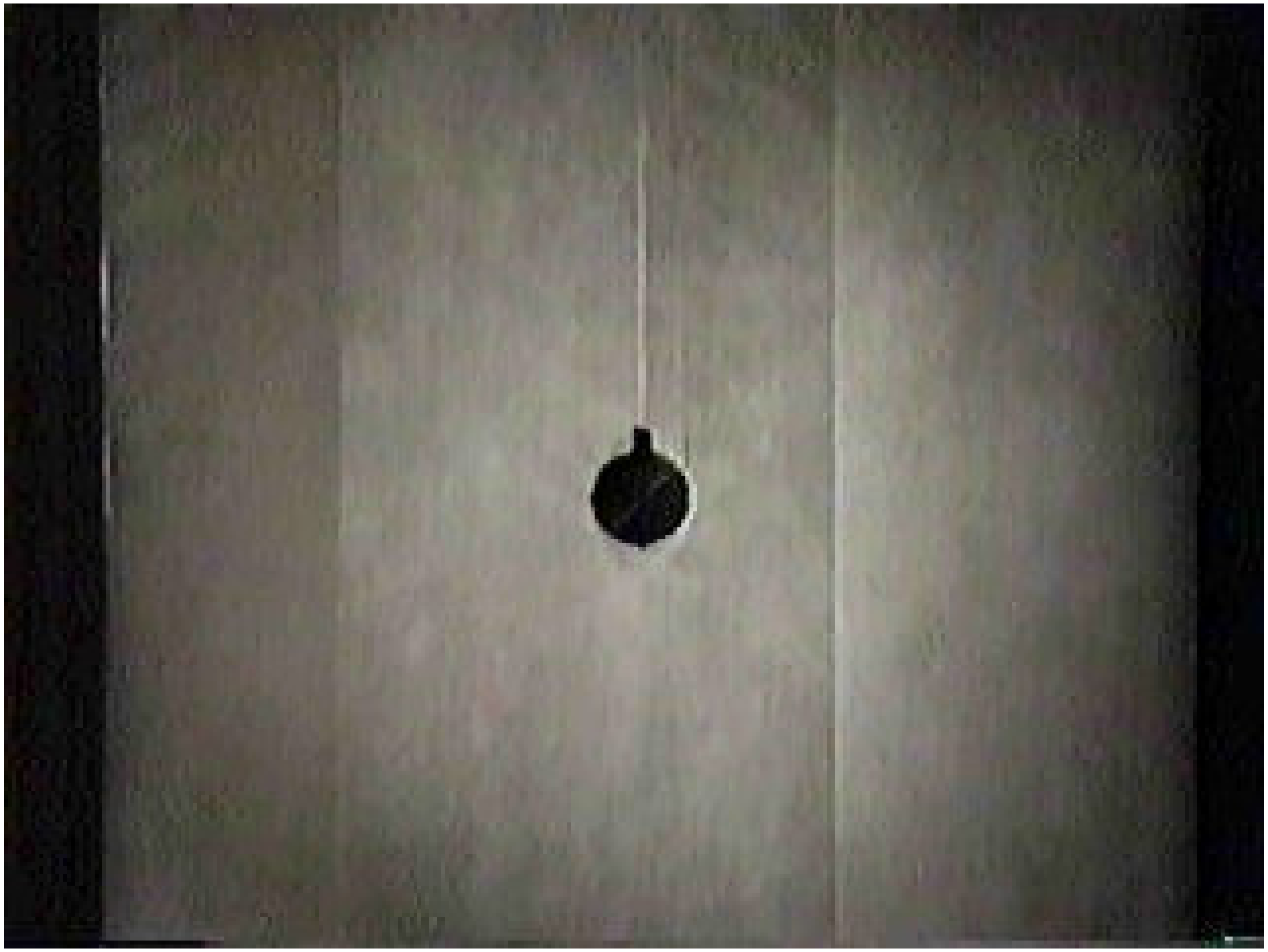


http://dennou-k.gaia.h.kyoto-u.ac.jp/library/gfd_exp/

Waves in shear
(slowly varying background state, varies on height scale $h \gg m^{-1}$)

$$\phi' == \mathrm{Re}\,\Phi(z)e^{ikx} = \mathrm{Re}\,\Phi_0(z)\exp(\frac{z}{2H})\exp[i(kx+mz-\omega t)]$$

$\Phi(z)$ slowly varying $[m|\Phi_0| \gg |d\Phi_0/dz|]$. $m = m(z)$, also slowly varying.

$$-i\omega U + ik\Phi = 0 \quad \rightarrow U = \frac{k}{\omega}\Phi = \frac{k}{\omega}\Phi_0 e^{z/2H}e^{imz}$$

$$ikU + \frac{1}{\rho}\frac{d}{dz}(\rho W) = 0$$

$$-i\omega\Theta + W\frac{d\theta_0}{dz} = 0 \quad \rightarrow W = \frac{i\omega}{d\theta_0/dz}\Theta = \frac{i\omega}{N^2}\left(\frac{1}{2H} + im\right)\Phi_0 e^{z/2H}e^{imz}$$

$$\frac{d\Phi}{dz} - \frac{\kappa}{H}\Pi\Theta = 0 \quad \rightarrow \Theta = \frac{H}{\kappa\Pi}\frac{d\Phi}{dz} = \frac{H}{\kappa\Pi}\left(\frac{1}{2H} + im\right)\Phi_0 e^{z/2H}e^{imz}$$

$$\overline{u'w'} = \frac{1}{2}\mathrm{Re}(UW^*) = -\frac{km}{2N^2}|\Phi_0|^2 e^{z/H}$$

Waves in shear
(slowly varying background state, varies on height scale $h \gg m^{-1}$)

$$\phi' == \mathrm{Re}\,\Phi(z)e^{ikx} = \mathrm{Re}\,\Phi_0(z)\exp(\frac{z}{2H})\exp[i(kx+mz-\omega t)]$$

$\Phi(z)$ slowly varying $[m|\Phi_0| \gg |d\Phi_0/dz|]$. $m = m(z)$, also slowly varying.

Momentum flux is constant: (we'll see this later)

$$F_0 = \rho\overline{u'w'} = -\frac{1}{2}\rho_0\frac{km(z)}{N^2(z)}|\Phi_0(z)|^2$$

$$\rightarrow |\Phi_0(z)|^2 = -2\frac{F_0}{\rho_0 k}\frac{N^2(z)}{m(z)}$$

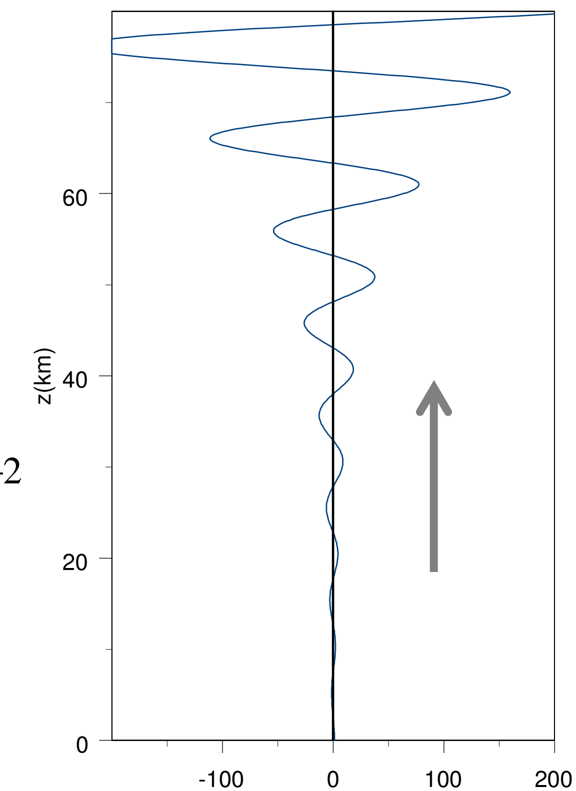
so

$$\phi' = \left(\frac{2F_0}{\rho_0}\right)^{\frac{1}{2}}\mathrm{Re}\left[\frac{N^2(z)}{k\,|m(z)|}\right]^{\frac{1}{2}}\exp(\frac{z}{2H})\exp[i(kx+mz-\omega t)]$$

varying mean state — density factor (usually dominates)

$$\phi' = \left(\frac{2F_0}{\rho_0}\right)^{\frac{1}{2}} \mathrm{Re}\left[\frac{N^2(z)}{k\,|m(z)|}\right]^{\frac{1}{2}} \exp\left(\frac{z}{2H}\right)\exp[i(kx + mz - \omega t)]$$

$$c_{gz} = \mp\frac{km}{N^2}(\bar{u} - c)^3 \simeq \frac{k}{N}(c - \bar{u})^2$$



Typical values:

$$2\pi/k = 500\mathrm{km},\ c - \bar{u} = 30\mathrm{ms}^{-1},\ N^2 = 4\times10^{-4}\mathrm{s}^{-2}$$

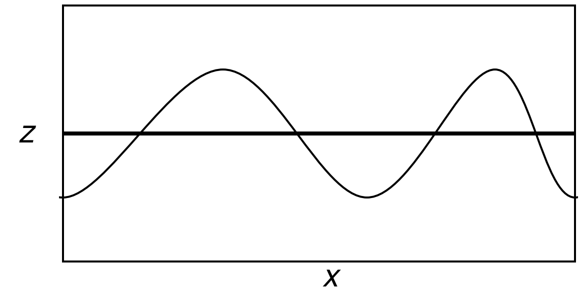
$$c_{g,z} \simeq 5ms^{-1}$$

→ 0 to 100 km in $20000s \simeq 6$ hr

→ weakly dissipated

(iii) momentum transport

Zonal Means



Define (Eulerian) zonal mean for $a(x, y, z, t)$:
[periodic in x: $a(x+L, y, z, t) = a(x, y, z, t)$]

$$\bar{a}(y, z, t) = \frac{1}{L}\int_0^L a(x, y, z, t)\, dx$$

eddy (wave) component

$$a'(x, y, z, t) = a(x, y, z, t) - \bar{a}(y, z, t)$$

by definition

$$\overline{a'} = 0 \;; \qquad \overline{\left(\frac{\partial a}{\partial x}\right)} = 0 \;:$$

$$\overline{\left(\frac{\partial a}{\partial [y, z, t]}\right)} = \frac{\partial \bar{a}}{\partial [y, z, t]}$$

$$\overline{a\,\frac{\partial b}{\partial x}} = \overline{\left(\frac{\partial}{\partial x} ab\right)} - \overline{b\,\frac{\partial a}{\partial x}} = -\overline{b\,\frac{\partial a}{\partial x}}$$

Action of waves on the mean state

Mean momentum eq.:

$$\left(\overline{u' \frac{\partial u'}{\partial x}} = \frac{1}{2}\overline{\frac{\partial u'^2}{\partial x}} = 0 \right)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{w}\frac{\partial \bar{u}}{\partial z} = \bar{G} - \overline{u'\frac{\partial u'}{\partial x}} - \overline{w'\frac{\partial u'}{\partial z}}$$

$$= \bar{G} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{u'w'}\right) + \overline{u'\left(\frac{\partial u'}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial z}(\rho w')\right)}$$

$$= \bar{G} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{u'w'}\right)$$

Mean continuity eq.:

$$\overline{\frac{\partial u}{\partial x}} + \overline{\frac{1}{\rho}\frac{\partial}{\partial z}(\rho w)} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho\bar{w}) = 0$$

→ $\bar{w} = 0$ everywhere, if zero on z =0 and

$$\frac{\partial \bar{u}}{\partial t} = \bar{G} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{u'w'}\right)$$

Similarly,

$$\frac{\partial \bar{\theta}}{\partial t} = (\rho\Pi)^{-1}\bar{J} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'\theta'}\right)$$

→ eddy fluxes of momentum, $\rho\overline{u'w'}$, and heat $\rho\overline{w'\theta'}$.

Eddy fluxes for steady, inviscid, adiabatic waves in shear

linearized equations

$$\frac{\partial u'}{\partial t} + u_0 \frac{\partial u'}{\partial x} + w' \frac{\partial u_0}{\partial z} + \frac{\partial \phi'}{\partial x} = G'$$

$$\frac{\partial u'}{\partial x} + \frac{1}{\rho} \frac{\partial (\rho w')}{\partial z} = 0$$

$$\frac{\partial \theta'}{\partial t} + u_0 \frac{\partial \theta'}{\partial x} + w' \frac{\partial \theta_0}{\partial z} = (\rho \Pi)^{-1} J'$$

$$\frac{\partial \phi'}{\partial z} - \frac{\kappa}{H} \tilde{\Pi}\, \theta' = 0$$

(1) eddy heat flux

Multiply 3rd eq. by θ' and average:

$$\overline{\theta' \frac{\partial \theta'}{\partial t}} + u_0 \overline{\theta' \frac{\partial \theta'}{\partial x}} + \overline{w'\theta'} \frac{\partial \theta_0}{\partial z} = \overline{\theta' J'}$$

But $\overline{\theta' \partial \theta' / \partial x} = \frac{1}{2} \overline{\partial \theta'^2 / \partial x} = 0$; if wave *amplitudes* are steady, $\overline{\theta'^2}$ is steady in time, for *adiabatic* eddies ($J' = 0$) then,

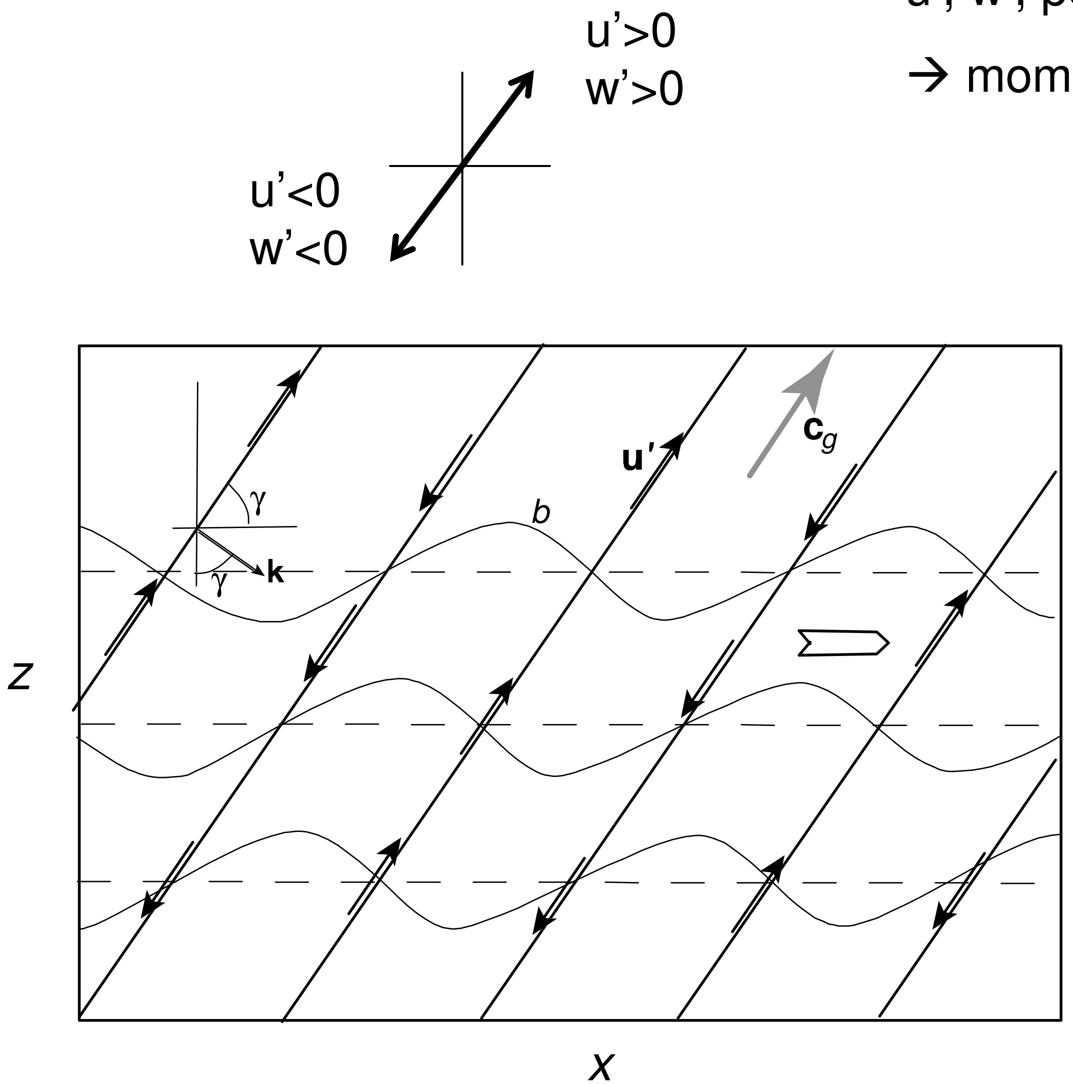
$$\boxed{\overline{w'\theta'} = 0}$$

→ steady, adiabatic ($J' = 0$) waves have zero vertical heat flux.

If phase tilt is as shown:

u’, w’, positively correlated

→ momentum flux > 0



Momentum flux for steady, conservative ($G' = J' = 0$) waves (detailed derivation)

First take mean of $u' \times$ eddy momentum equation:

$$\overline{u'\frac{\partial u'}{\partial t}} + u_0\overline{u'\frac{\partial u'}{\partial x}} + \overline{u'w'}\frac{\partial u_0}{\partial z} + \overline{u'\frac{\partial \phi'}{\partial x}} = \overline{u'G'}$$

$$\rightarrow \qquad \overline{u'w'}\frac{\partial u_0}{\partial z} + \overline{u'\frac{\partial \phi'}{\partial x}} = 0$$

for steady *conservative* waves. But

$$\overline{u'\frac{\partial \phi'}{\partial x}} = \overline{\frac{\partial}{\partial x}(u'\phi')} - \overline{\phi'\frac{\partial u'}{\partial x}} = \overline{\frac{1}{\rho}\phi'\frac{\partial}{\partial z}(\rho w')}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'\phi'}\right) - \overline{w'\frac{\partial \phi'}{\partial z}}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'\phi'}\right) + \frac{\kappa}{H}\Pi\,\overline{w'\theta'}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'\phi'}\right)$$

$$\rightarrow \rho\overline{u'w'}\frac{\partial u_0}{\partial z} + \frac{\partial}{\partial z}\left(\rho\overline{w'\phi'}\right) = 0$$

for steady, conservative waves.

$$\frac{\partial u'}{\partial x} + \frac{1}{\rho}\frac{\partial(\rho w')}{\partial z} = 0$$

From continuity, define streamfunction ξ such that

$$w' = -\frac{\partial \xi'}{\partial x} \; ; u' = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho \xi') \; .$$

Then write momentum eq. (since $\partial/\partial t = -c \, \partial/\delta x$)

$$(\bar{u} - c)\frac{\partial u'}{\partial x} + \frac{du}{dz}\frac{\partial \xi'}{\partial x} = -\frac{\partial \phi'}{\partial x}$$

$$\rightarrow (\bar{u} - c)u' + \frac{du}{dz}\xi' = -\phi'$$

But

$$\overline{w'\xi'} = -\overline{\xi'\frac{\partial \xi'}{\partial x}} = 0$$

so

$$(\bar{u} - c)\overline{u'w'} = -\overline{w'\phi'}$$

and

$$\boxed{(u - c)\frac{\partial}{\partial z}\left\langle \rho \overline{u'w'} \right\rangle = 0}$$

Summary

steady, adiabatic, inviscid, waves ($\bar{u} \neq c$):

$$\overline{w'\theta'} = 0 \;\; ; \quad \frac{\partial}{\partial z}\left(\rho\overline{u'w'}\right) = 0$$

momentum flux is constant — manifestation of *wave activity* conservation.
[NB: $\partial\left(\rho\overline{w'\phi'}\right)/\partial z \neq 0$, if $\partial\bar{u}/\partial z \neq 0$ $\rightarrow$ "energy flux" not constant]

Forcing of mean state:

$$\frac{\partial\bar{u}}{\partial t} = \bar{G} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{u'w'}\right)$$
$$\frac{\partial\bar{\theta}}{\partial t} = (\rho\Pi)^{-1}\bar{J} - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'\theta'}\right)$$

special case of the *nonacceleration theorem*:
mean flow is indifferent to the presence of steady, conservative waves
(unless waves influence $\bar{G}$, $\bar{J}$).

Sign of the momentum flux

$$c_0 = \frac{\omega}{k} = \pm N\left(m^2 + \frac{1}{4H^2}\right)^{-1/2}$$

add mean flow $\bar{u}$:

$$c = c_0 + \bar{u} = \bar{u} \pm N\left(m^2 + \frac{1}{4H^2}\right)^{-1/2}$$

$$c_{gz} = k\frac{\partial c}{\partial m} = \mp Nkm\left(m^2 + \frac{1}{4H^2}\right)^{-3/2} = \mp\frac{km}{N^2}(c - \bar{u})^3$$

Upward propagating wave: $c_{gz} > 0 \rightarrow sgn(km) = sgn(\bar{u} - c)$.

$$\phi' = \mathrm{Re}\,\Phi(z)\exp(\frac{z}{2H})\exp[i(kx + mz - \omega t)]$$

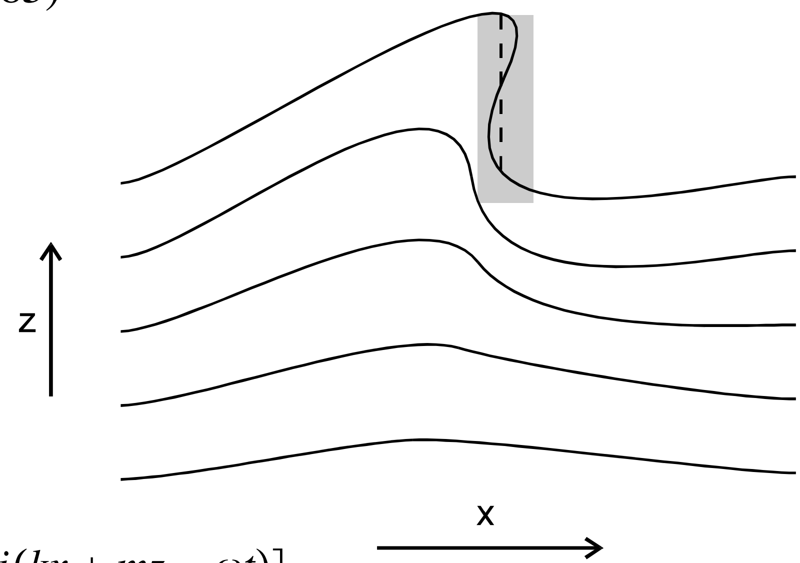
$$\rightarrow \rho\overline{u'w'} = -\frac{1}{2}\rho_0\frac{km}{N^2}|\Phi(z)|^2$$

$$\rightarrow sgn(\rho\overline{u'w'}) = -sgn(km) = sgn(c - \bar{u})$$

→ momentum flux is nonzero for $m \neq 0$, and its sign is that of $c - u$ ("pseudomomentum rule")

(iv) internal gravity wave breaking

Gravity wave breaking (of the simplest kind)
(Lindzen, JGR, 86, 9707, 1981; JAS, 42, 301, 1985)



Wave breaks by convective instability where

$$\frac{\partial\theta}{\partial z} = \frac{\partial\bar{\theta}}{\partial z} + \frac{\partial\theta'}{\partial z} = \frac{\partial\bar{\theta}}{\partial z}\left[1 + \frac{\partial\theta'/\partial z}{\partial\bar{\theta}/\partial z}\right] < 0$$

$$\phi' = \mathrm{Re}\left(\frac{2F_0}{\rho_0 k}\right)^{\frac{1}{2}}\left[-\frac{N^2(z)}{m(z)}\right]^{\frac{1}{2}} \exp\left(\frac{z}{2H}\right)\exp[i(kx + mz - \omega t)]$$

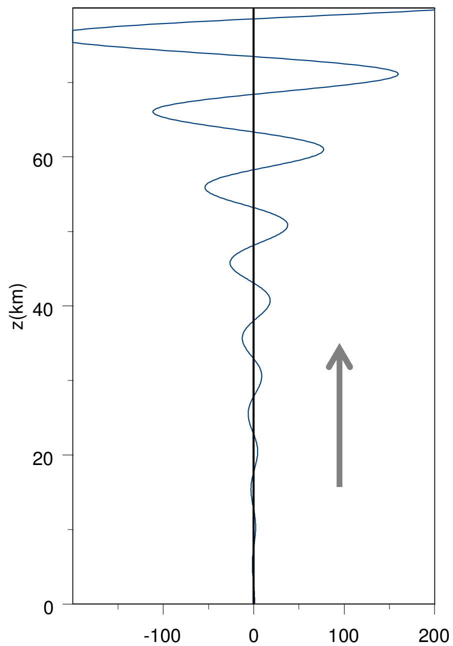
$$\frac{\partial\phi'}{\partial z} - \frac{\kappa}{H}\Pi\theta' = 0$$

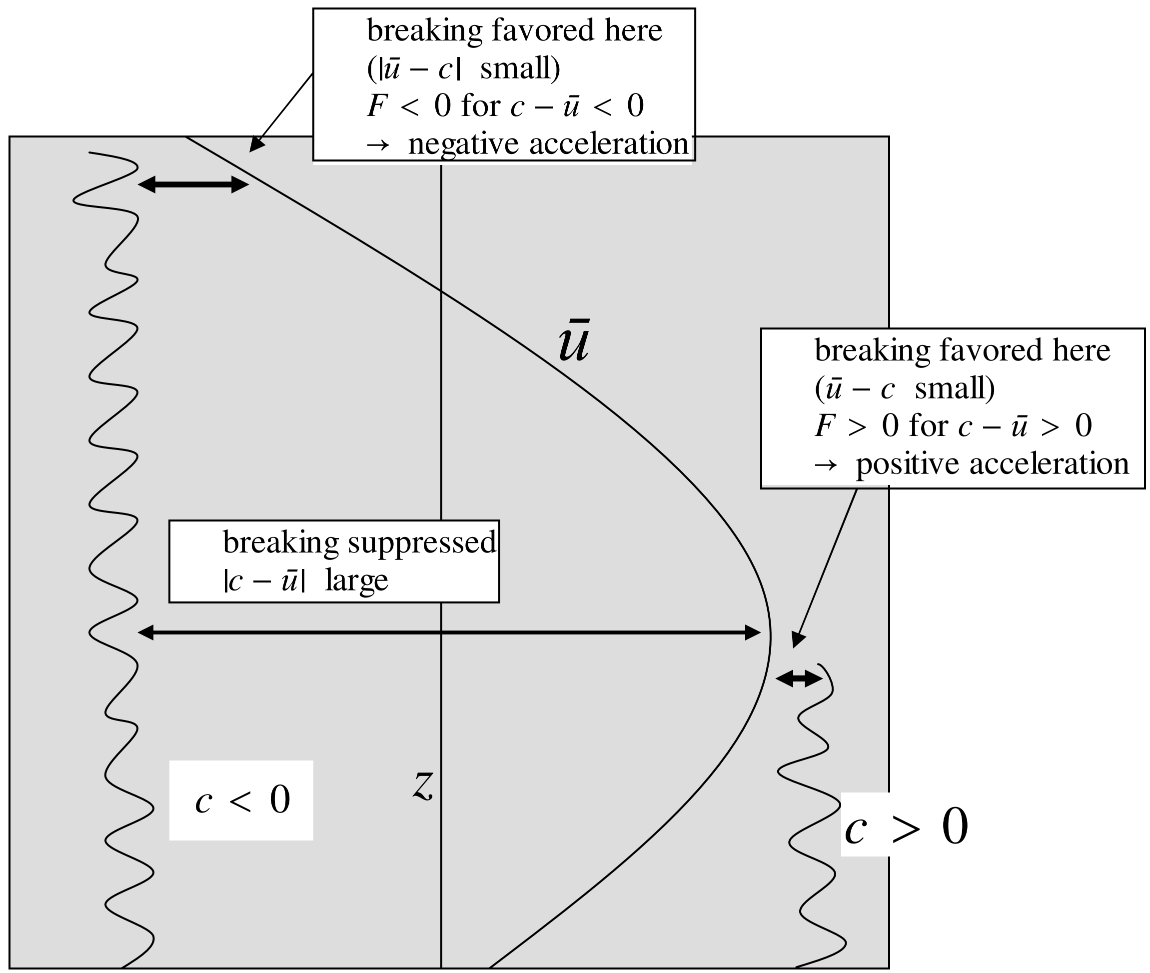
$$\frac{\partial\theta'}{\partial z} \simeq \mathrm{Re}\left(\frac{2F_0}{\rho_0 k}\right)^{\frac{1}{2}} \frac{HN^{5/2}}{\kappa\Pi(c-\bar{u})^2\sqrt{-m(z)}} \exp\left[\frac{z}{2H}\right]\exp[i(kx + mz - \omega t)]$$

$$\left|\frac{\partial\theta'/\partial z}{\partial\bar{\theta}/\partial z}\right| = \left(\frac{2F_0}{\rho_0 k}\right)^{\frac{1}{2}} \frac{N}{(c-\bar{u})^2\sqrt{-m(z)}} e^{z/2H} \sim \sqrt{\frac{N}{(c-\bar{u})^3}}\; e^{z/2H} \quad \left(\text{for } m^2 \gg \frac{1}{4H^2}\right)$$

— breaking favored at large z and/or small $|c - \bar{u}|$

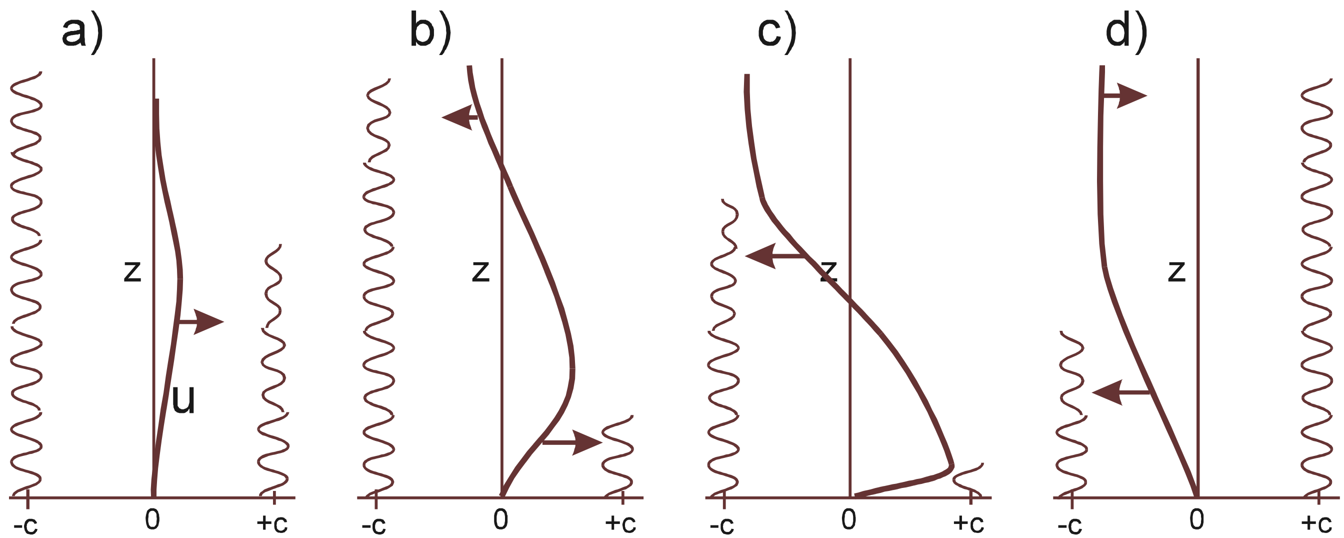
— breaking favored at large z and/or small $|c - \bar{u}|$





→ Internal gravity wave breaking can *reinforce* zonal flow
(we'll see importance of this later)

Oscillating mean flow can be produced by two upward propagating waves of opposite zonal phase speed:



“QBO” in the lab

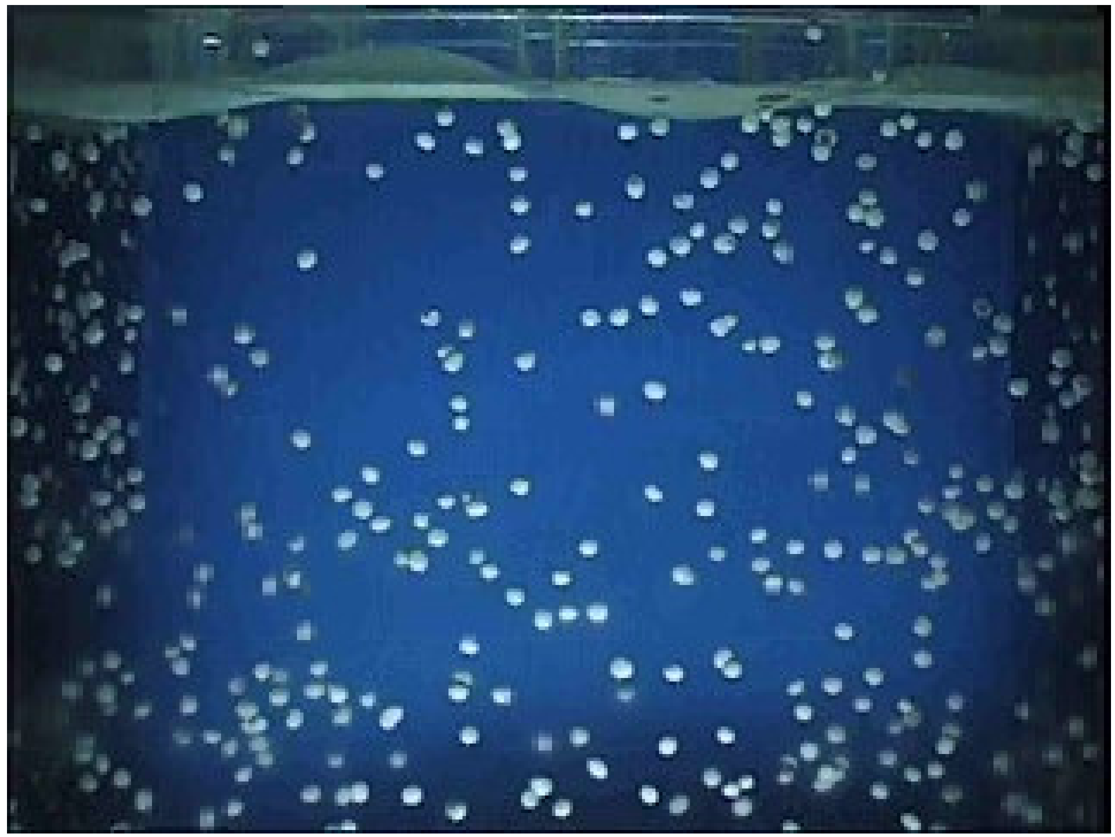
subcritical forcing

実験室の中の空と海

“QBO” in the lab

Atmosphere and Ocean in a Laboratory

supercritical forcing



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