Anelastic spherical dynamos with variable conductivity

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Planetary magnetic fields

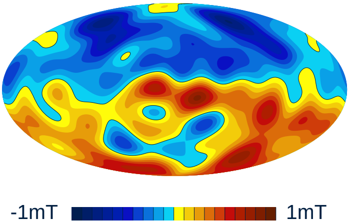
Magnetic fields have been mapped for Mercury, Earth, Jupiter, Saturn, Uranus and Neptune.

We know Ganymede has an active dynamo, and Mars had a dynamo which switched off 350Myr after formation.

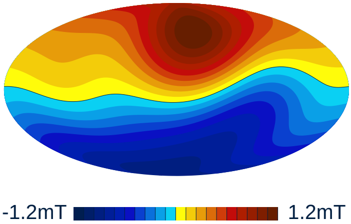
It has been suggested that the minor planet Vesta may have had a dynamo in the past.

Likely to be dynamos in extra-solar planets, e.g. hot Jupiters.

The great diversity of planetary magnetic fields suggests that the physical conditions in their interiors varies considerably.



Earth's B_r at CMB



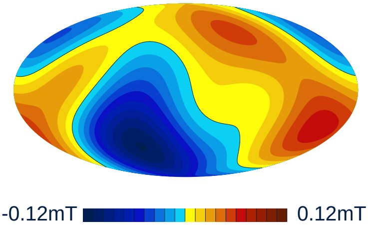
Jupiter's B_r at surface

Pre-Juno Jupiter magnetic field went up to spherical harmonic degree 4 reliably. More Earth-like at high resolution?

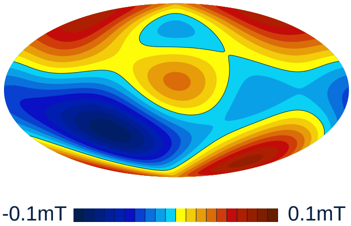
Both fields are broadly dipolar, with axis offset by $\sim 10^\circ$.

Both planets are believed to be convecting in their electrically conducting zones. Earth may have a thin stable layer just below CMB.

Magnetic fields of the Ice Giants



Uranus B_r at surface



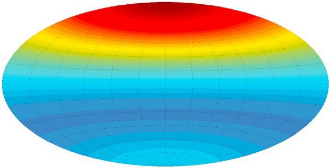
Neptune B_r at surface.

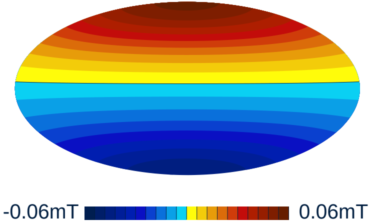
These magnetic fields are not mainly dipolar, they are a mixture of non-axisymmetric quadrupolar and dipolar modes.

Fields are significantly weaker, but we don't know much about the internal structure of these planets.

It may be that the dynamo region is near the surface but the fields there are really weaker than in the geodynamo, or it may be the fields are generated in the deep interior and are strong there.

Magnetic fields of Mercury and Saturn





Field strength $\sim$ 1% Earth's

Mercury B_r at surface

Saturn B_r at surface.

These magnetic fields are quite axisymmetric, though Mercury's field is equatorially asymmetric.

Dynamo generated fields must be non-axisymmetric, but they can appear axisymmetric at the surface.

It has been suggested that these planets have a stably stratified layer above the dynamo region. If this layer is electrically conducting and differentially rotating, the non-axisymmetric components in the dynamo region are filtered out by the stably stratified layer.

Juno and Cassini Grand Finale

Juno is gathering gravity and magnetic field data from Jupiter.

Mission will last longer than planned: close flyby occurs every 53 days.

Gravity data may indicate the size of the core, and the internal rotation rate, hence how deep the zonal flows are.

Magnetic data looks very promising, and has already raised new questions.

Cassini entered Saturn in September, and detected the non-axisymmetric field component for the first time.

Saturn's axisymmetric magnetic field

• Saturn's field is remarkably axisymmetic, dipole tilt is only about than 0.06° (Cao, 2017).

• Could be because the metallic hydrogen region is smaller than Jupiter's, because smaller mass means 2 Mbar pressure occurs deeper.

• Saturn has significantly less helium in its atmosphere compared to Jupiter and the Sun. Helium might have 'rained out'.

• Rain-out could (but not necessarily) lead to a stably stratified zone above the dynamo region.

• Here we examine whether Saturn-like almost axisymmetric fields can be generated solely because the metallic hydrogen layer is very deep.

Magnetic field outside the electrically conducting region is potential,

$$V = R_s \sum_{n=1}^{\infty} \sum_{m=0}^{m=n} \left(\frac{R_s}{r} \right)^{n+1} P_n^m(\cos\theta)(g_n^m \cos m\phi + h_n^m \sin m\phi) \tag{2.1}$$

where $\mathbf{B} = -\nabla V$, and r, θ and ϕ are spherical polar coordinates, $r = R_s$ is the planetary surface.

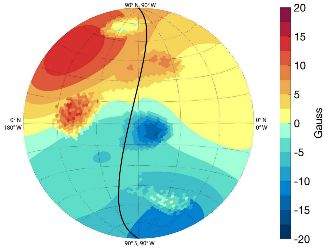
Determine g_n^m, h_n^m at $r = R_{obs}$, and evaluate field at top of the dynamo region $r = R_{dyn}$.

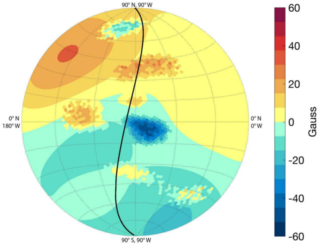
Attenuation $\sim (R_{dyn}/R_{obs})^{n+1} \sim 0.8^{n+1}$ since Juno passes only 4000 km above Jupiter's surface.

For Earth, attenuation is 0.546^{n+1}, so components with $n >\sim 13$ are obscured by crustal magnetism.

Jupiter's field can be seen at high resolution: may well change our view of geomagnetic field too.

Jupiter's field post Juno





Jupiter's B_r at the surface Jupiter's B_r at $r = 0.85R_{jup}$

The solid line is Juno's fly-by path. New model constructed by superimposing the new data on the old Jupiter model.

Only the field close to the Juno path can be reliably estimated.

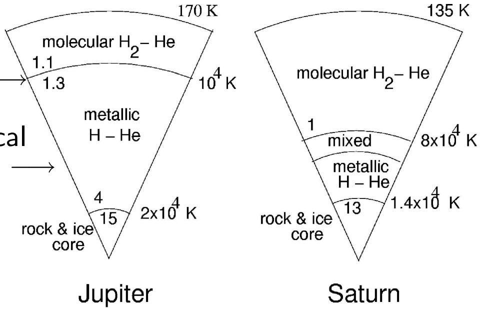
Revised field has intense flux spots, some near the equator, some near the poles.

Standard Model assumes Jupiter has rocky inner core of about 1% of the total mass of the planet. The French et al. 2012 model has radius ratio 0.0923. Based on computational quantum density functional techniques. Dynamo model extends from core to 3000km below the surface, the cut-off being for resolution reasons.

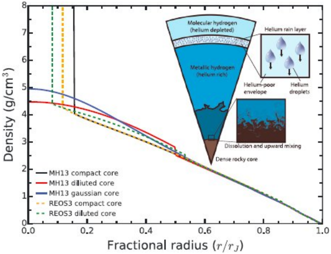
Low electrical conductivity ⟶

Transition zone ⟶

High electrical conductivity ⟶



Dilute core model?

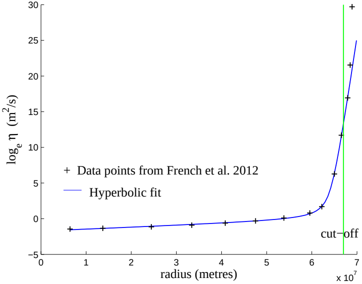


Wahl et al. (2017)

The standard French et al. model used for dynamo simulations fits the new Juno gravity data results rather well.

However, different density functional theories give different equations of state. Using one of these would allow a different equilibrium structure model that still satisfied Juno constraints.

Helium rain-out might have led to a dilute core model: also might have given rise to the stable layer suggested by Saturn's axisymmetric field.



Diffusivity $\eta = 1/\mu\sigma$, σ being the electrical conductivity.

$+$ signs are the French et al. 2012 model, the curve is a smoothed hyperbolic fit. The French et al. conductivity drops off super-exponentially beyond a radius of $0.85R_{jup}$. By $0.95R_{jup}$ the magnetic Reynolds number is less than 1.

Verifying the electrical conductivity profile

As more perijove passes are logged, Lowes-Mauersberger spectrum will be determined:

$$R_n = \left(\frac{R_s}{r}\right)^{2n+4} (n+1) \sum_{m=0}^{n} \left[\left(g_n^m\right)^2 + \left(h_n^m\right)^2 \right]$$

The slope of R_n with n is expected to be flat (white noise) when $r = R_{dyn}$; this gives a method of checking whether this occurs where the DFT based profiles predict.

Alternatively, compare the Juno spectrum with a dynamo model simulation. If the slope is correct, that will confirm that the correct conductivity profile is being used in the simulation.

As the extended mission evolves, it may be possible to detect changes in the Gauss coefficients: the secular variation. There is some evidence Jupiter's field does change slowly with time. Important as it gives independent evidence of convective velocity.

Mass conservation: $\nabla \cdot \bar{\rho}\mathbf{u} = 0$. The equation of motion is

$$\frac{1}{Pm}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla\frac{\hat{p}}{E} - \frac{2}{E}\hat{\mathbf{z}}\times\mathbf{u} + \frac{1}{E\bar{\rho}}(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$+\mathbf{F}_\nu - \frac{Pm}{Pr}\,RaS\frac{d\bar{T}}{dr}\hat{\mathbf{r}}.$$

The entropy equation is

$$\frac{\mathcal{D}S}{\mathcal{D}t} = \frac{Pm}{Pr}\left(\frac{1}{\bar{\rho}\bar{T}}\nabla\cdot\bar{\rho}\bar{T}\nabla S + H\right) + \frac{Pr}{PmRa\bar{T}}\left[\frac{\bar{\eta}}{E\bar{\rho}}(\nabla\times\mathbf{B})^2 + Q_\nu\right],$$

assuming constant kinematic entropy diffusivity and constant kinematic viscosity throughout the shell. Here PmH/Pr is the source term from the gradual cooling of Jupiter.

Dimensionless parameters

Input Parameters:

Ekman number $E = \nu/\Omega d^2$, Rayleigh number $Ra = \dfrac{\Delta S T_c d^2}{\nu\kappa}$.

Prandtl number $Pr = \nu/\kappa$, Magnetic Prandtl number $Pm = \nu/\eta_c$, η magnetic diffusivity, ν kinematic viscosity, κ is the entropy diffusivity, $d = r_o - r_i$.

ΔS is the imposed entropy drop across the shell. T_c and η_c are values at $r = r_i + d/2$.

Inner boundary no-slip, outer boundary stress-free. Electrically insulating bcs. Mostly fixed entropy, some fixed flux bcs.

Outputs: Rossby number $Ro = U/\Omega d$, Magnetic Reynolds number, $Rm = Ud/\eta$, Elsasser number $B^2/\mu\Omega\rho\eta$, magnetic and kinetic energies, zonal flow, heat flux, dipole and quadrupole moments.

Choice of units and diffusivities

$U^* \sim 10^{-2}$ m/sec fits the secular variation data and the requirement to get the heat flux out. With French et al. diffusivity, $Rm \sim 10^6$.

Simulations only possible for Rm up to 5×10^3, so magnetic diffusivity must be enhanced. If the typical dimensionless velocity 10^3 corresponds to 10^{-2} m/sec, the magnetic diffusion time is then $\sim 200,000$ years.

Unit of field strength is $(\rho_c \mu \Omega \eta_c)^{1/2} \sim 3$ gauss, too low: use the enhanced magnetic diffusion? Best to interpret field strength by matching the model to the known surface value.

Thermal and viscous diffusivities much larger than their molecular values. Inevitable, to cut off the turbulent spectrum before it reaches very small scales.

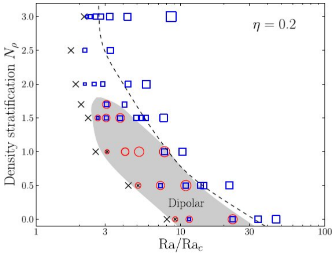
Parameter space of Anelastic dynamos

There are severe computational constraints on compressible dynamos. Dynamo action can only be established by integrating for over a diffusion time. Rm must generally be of order 10^3 for dynamo action, so integration for many turn-over times is needed.

Low E is desirable, but less than 10^{-5} currently not practical. Ra must be large enough to give vigorous convection, but not too large to destroy the dominance of the Coriolis force. Window of acceptable Ra not that great at feasible E.

Pr not too severely constrained. Low Pr more realistic. Astrophysical value of Pm should be less than unity. Larger Pm more computationally attractive, as it allows large Rm without large Reynolds number Re. Also gives stronger field. However, too large Pm leads to smaller timesteps, so a compromise is best.

Steady Dipoles not easy to find



Taken from Gastine et al. 2012. Red circles dipole dominated solutions, blue squares multipolar solutions. N_ρ is number of scale heights in the polytrope. No dipole dominated solutions at all at large N_ρ, $E = 10^{-4}$ for constant conductivity model.

Zonal flows in the outer regions (stress-free boundaries), dynamo becomes $\alpha\omega$ and dynamo waves break up steady dipoles.

Situation improves somewhat if variable conductivity is used, as dynamo action doesn't occur where zonal flow is strongest. But zonal flow usually penetrates somewhat into dynamo region.

If dynamo has a dynamically strong field, it can suppress the differential rotation, and hence stay dipole dominant. Bistability is an issue.

Variety of Dynamos in anelastic models

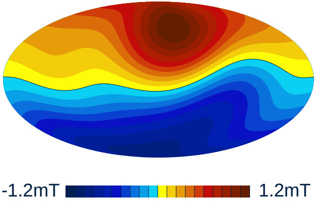
- Many different types of dynamo found.

- Hemispherical dynamos, quadrupolar dynamos, steady and reversing dipolar solutions and small-scale dynamos.

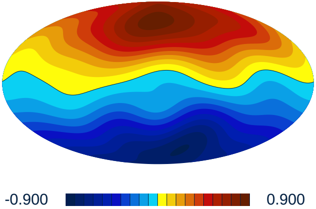
- The stress-free outer boundary helps generate a wider range of dynamos in Boussinesq models (Simitev and Busse), so this is a partial explanation.

- However, there does seem to be a greater diversity with anelastic models. The steady dipole window in parameter space is much reduced.

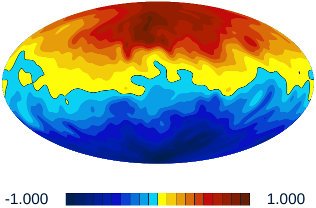
$Pr = 0.1$, $E = 2.5 \times 10^{-5}$ solution snapshot



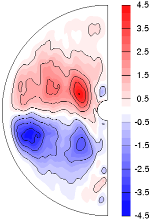
Jupiter surface radial B_r



Model radial B_r snapshot truncated at harmonic $\ell \leq 5$



Model radial B_r (full resolution)



Axisymmetric meridional B_ϕ

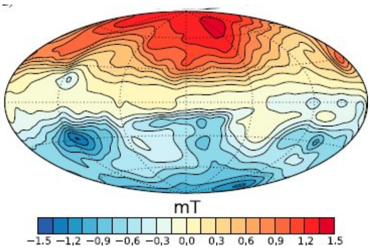
How do we get the dipolar solutions?

With a uniform entropy source, expected if Jupiter is convecting normally everywhere outside the small core, only found steady dipoles at small $Pr \sim 0.1$

Also need $Pm \sim 3$ (or bigger) because field has to be strong enough to suppress differential rotation.

Domain of instability might increase at lower E, but very expensive to go there.

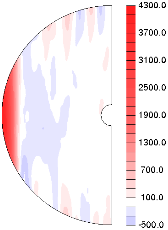
Jupiter's dynamo might be reversing: expected timescale thousands of years. But dynamo wave snapshots don't look very Jupiter-like.



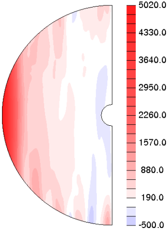
Taken from Gastine et al 2014. $Pr = 1$, $Pm = 0.6$, $E = 10^{-5}$. The driving is basal heating, coming out of the core. This is perhaps less realistic, but it makes it easier to get dipole dominated solutions, not necessary to go to low Pr.

Despite different parameters and different driving, the Göttingen models look quite similar to the Leeds models.

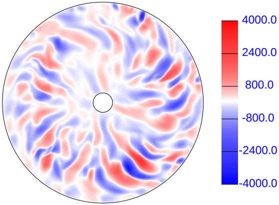
They do have more differential rotation at the top of the dynamo region, hence dynamo waves, but their deeper driving means that this doesn't disturb the deep dipole part of the dynamo.



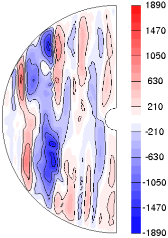
Axisymmetric part of u_ϕ run A



Axisymmetric part of u_ϕ run C

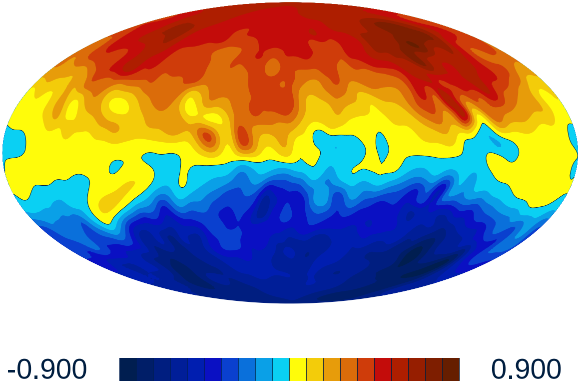


Equatorial section of u_r



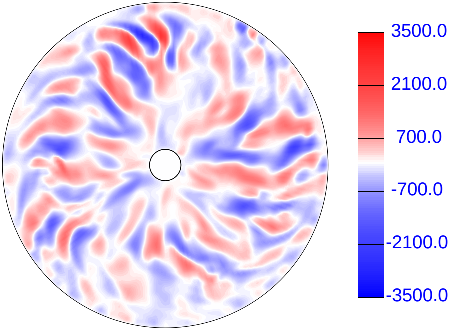
Random meridional slice of u_r

$Pr = 0.1$ Dynamo Movie

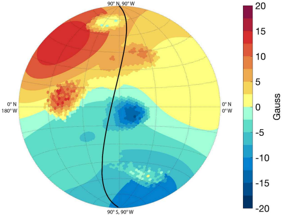


Time-lapse movie of B_r at the surface, for the $Pr = 0.1$ internal heated dynamo. Fixed entropy boundaries. $Ra = 1.2 \times 10^7$, $Pm = 3.0$, $E = 2.5 \times 10^{-5}$. Dynamo remains dipolar dominant.

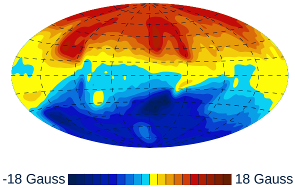
Flow Movie for the $Pr = 0.1$ Dynamo



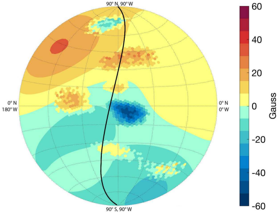
Time-lapse movie of u_r in the equatorial plane, for the $Pr = 0.1$ internal heated dynamo. Fixed entropy boundaries.
$Ra = 1.2 \times 10^7$, $Pm = 3.0$, $E = 2.5 \times 10^{-5}$. Note the difference between the magnetically locked interior and zonal flow dominated molecular region.



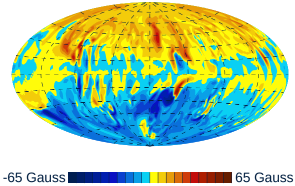
Juno field at $r = R_{jup}$



Dynamo simulation at $r = R_{jup}$

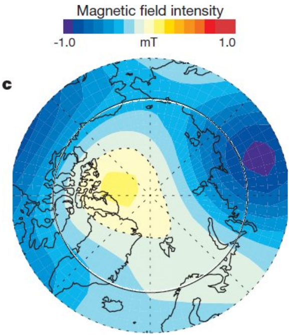


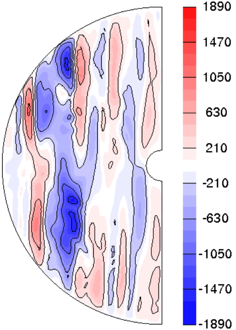
Juno field at $r = 0.85 R_{jup}$



Dynamo simulation at $r = 0.85 R_{jup}$

Weak polar field: signature of the core?





Weak polar patch at the N.pole
Olson and Aurnou (1999)

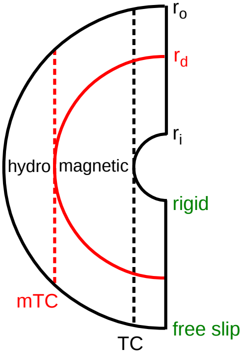
Jupiter model meridional section of u_r

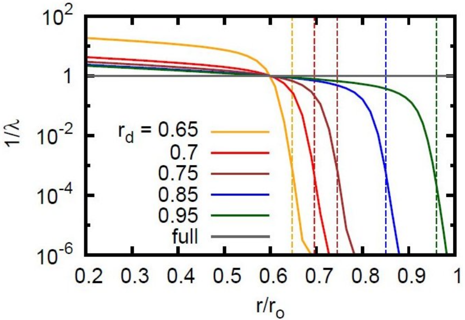
Geodynamo models suggest this is due to the inner core: convection rolls line up just outside the inner core equator.

Could the same thing happen in Jupiter if it has a larger core? Rolls less columnar in simulations, but *Ro* overestimated.

Jupiter dynamo models after Juno

- Anelastic dynamo models, convecting from the core to the surface, can produce Jupiter-like fields.
- The small scale features found by Juno were predicted by the models.
- Models with internal heating only give dipole dominated fields in restricted area of parameter space (low E, low Pr, moderate Pm).
- Equatorial flux spots and weak polar fields not common in existing models.
- Models with helium separation (dilute core) worth exploring.





Schematic model — Electrical conductivity of models tested

Conductivity $= 1/\lambda$, λ magnetic diffusivity. Saturn corresponds to $r_d = 0.65$, Jupiter to $r_d = 0.95$.

Chosen conductivity models are scaled Jupiter models (French et al. 2012). mTC is the magnetic tangent cylinder, where $r = r_d$ and $Rm(r) = 1$.

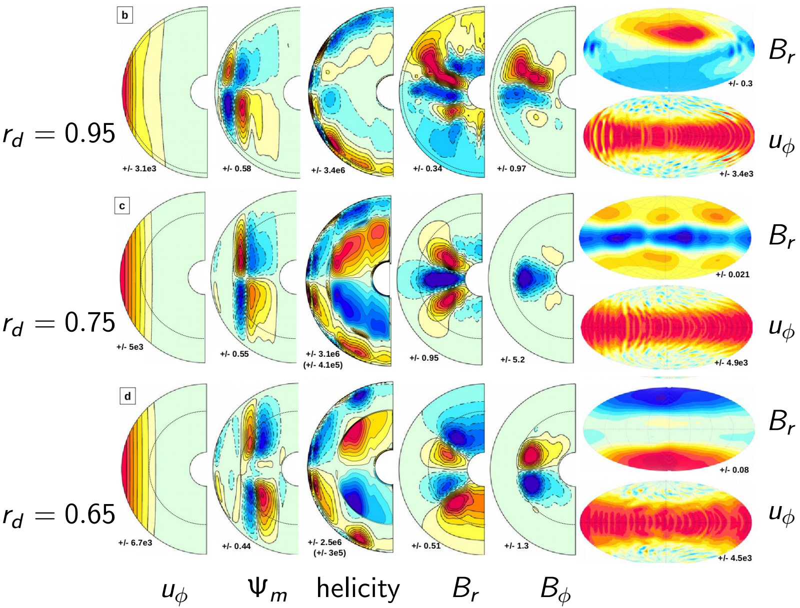
Anelastic dynamo model parameters

The standard rotating, convection-driven, anelastic dynamo equations are solved numerically.

Typical input Parameters: $E = 5 \times 10^{-5}$, $Ra = 10^7$, Prandtl number $Pr = 0.25$, Magnetic Prandtl number $Pm = 3$. Normalised at shell midpoint.

Inner boundary no-slip, outer boundary stress-free. Electrically insulating bcs. Outer boundary heat flux constant over surface. Most models are internally heated (Kelvin-Helmholtz contraction), some bottom-heated for comparison.

Simulations are run till they equilibrate, usually around a diffusion time (sometimes more). Lot of CPU consumed!


b
$r_d = 0.95$
+/- 3.1e3
+/- 0.58
+/- 3.4e6
+/- 0.34
+/- 0.97
+/- 0.3
+/- 3.4e3
B_r
u_ϕ
c
$r_d = 0.75$
+/- 5e3
+/- 0.55
+/- 3.1e6
(+/- 4.1e5)
+/- 0.95
+/- 5.2
+/- 0.021
+/- 4.9e3
B_r
u_ϕ
d
$r_d = 0.65$
+/- 6.7e3
+/- 0.44
+/- 2.5e6
(+/- 3e5)
+/- 0.51
+/- 1.3
+/- 0.08
+/- 4.5e3
B_r
u_ϕ
u_ϕ
Ψ_m
helicity
B_r
B_ϕ

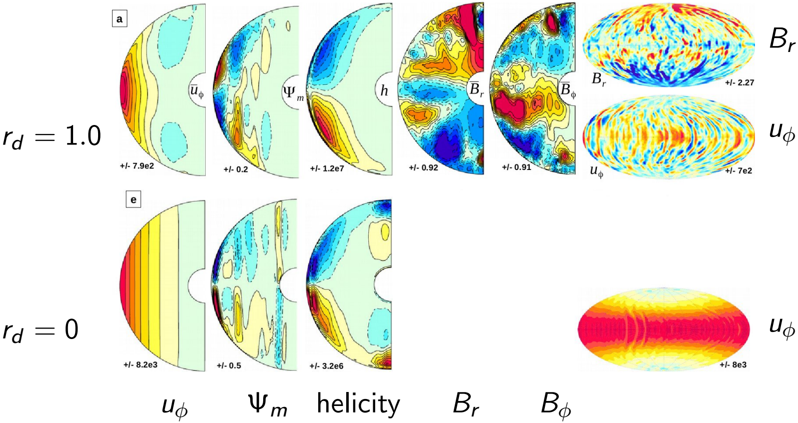
Key points from the results

• Zonal flow confined to region outside mTC. Deeper and extends to higher latitude in Saturn-like cases.

• Meridional circulation is driven by the magnetic field.

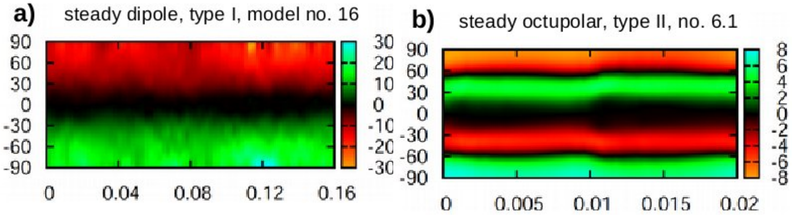
• Magnetic field reverses the sign of the helicity $h = \mathbf{u} \cdot \boldsymbol{\omega}$ (Sreenivasan + Jones 2011 found magnetic field effects helicity in Boussinesq models: larger effect in anelastic models).

• Models between the Jupiter and Saturn cases are mostly quadrupolar.



Top case has uniform electrical conductivity, bottom case is non-magnetic. As expected, top case has a weak dipole and quadrupole component. No helicity reversal in either case. Zonal flow strongest in non-magnetic case.

Types of dynamo found: dipole octupole

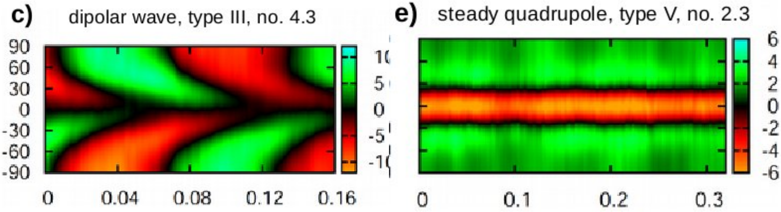


Radial B_r as a function of latitude, evolving with time (in diffusion times).

(a) is the Jupiter-like case, $r_d = 0.95$, which leads to a steady dipole (or one that reverses on a difusive time-scale).

(b) If r_d is decreased to 0.85, (planet of mass between Jupiter ands Saturn), dynamo generates a steady octupole, with positive and negative radial field in both hemispheres.

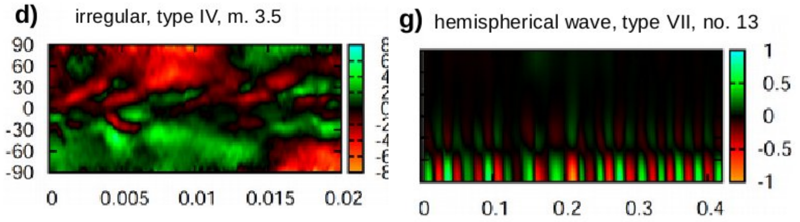
Dipolar waves, steady quadrupolar dynamo



Radial B_r as a function of latitude, evolving with time (in diffusion times).

(c) has Saturn-like r_d, $r_d = 0.65$. Fairly periodic dynamo waves, dipolar field that reverses on a difusive time-scale.

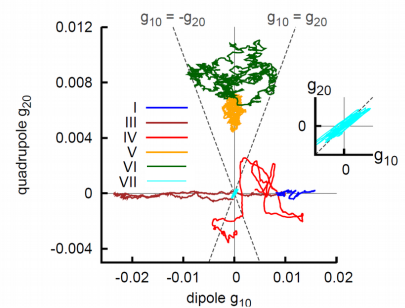
(e) Here $r_d = 0.75$ and the Rayleigh number is a little higher than in the octupolar (b). For intermediate r_d higher Rayleigh numbers lead to quadupolar dynamos, but very high Ra gives a small scale dynamo. Increasing Ra slightly makes the central belt oscillate sinuously (small dipole wave added).



Radial B_r as a function of latitude, evolving with time (in diffusion times).

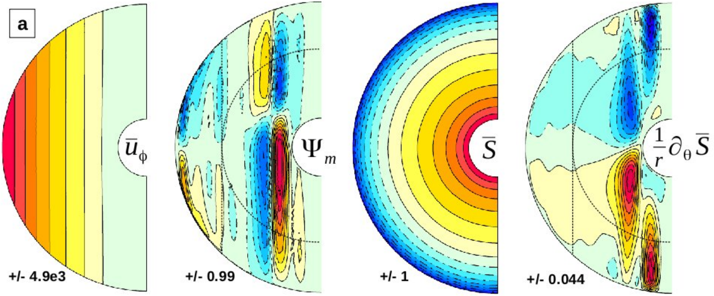
(d) is an irregular multipolar dynamo. This type of solution is universal if the Rayleigh number is high enough to make the Rossby number $O(1)$. To keep in the rapidly rotating regime we have to restrict Ra at fixed E.

(g) is a hemispherical dynamo wave. Here $r_d = 0.65$, $Ra = 5 \times 10^6$ and $Pr = 0.15$.



Dipole Gauss coefficient is $g_{10}(t)$, quadrupole coefficient is $g_{20}(t)$. Plot is the time-dependent phase diagram for the different dynamos.

I - Steady dipole: III - Dipolar wave: IV - Multipolar dynamo:
V - Steady quadrupole: VI - Wobbling quadrupole:
VII - Hemispherical dynamo.

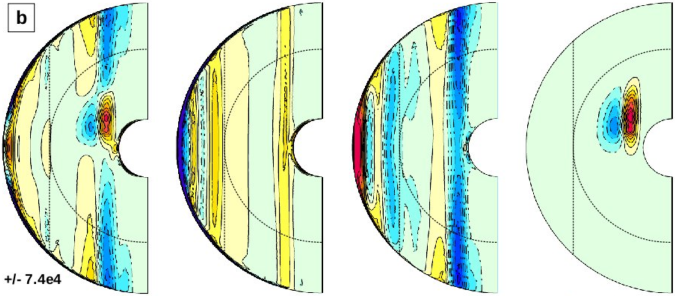


Time-averaged meridional sections of the hemispherical dynamo run.

Zonal flow $\bar{u}_\phi$, entropy $\bar{S}$ and the latitudinal gradient of the entropy are all rather symmetric about the equator.

Meridional circulation is not symmetric. This suggests that the asymmetric magnetic field is generating an asymmetric meridional circulation which maintains the asymmetric field.

Hemispherical dynamo dynamics (ii)



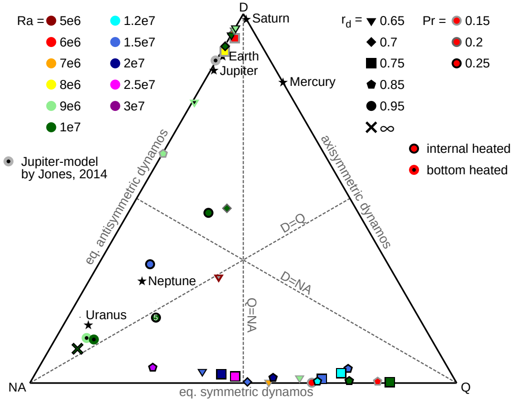
$$\overline{\rho}\,\overline{\boldsymbol{u}}_m \cdot \nabla L = \quad Pm\nabla \cdot \overline{\rho} s^2 \nabla \left(\frac{\overline{u}_\phi}{s}\right) \quad -\nabla \cdot \overline{\rho} s\, \overline{\boldsymbol{u}' u'_\phi} \quad + \frac{Pm}{E} \nabla \cdot \left(\frac{s}{\overline{\rho}} \overline{\boldsymbol{B} B_\phi}\right)$$

Time-averaged meridional sections of the angular momentum balance for the hemispherical dynamo run.

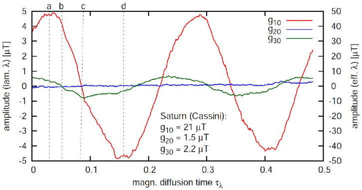
Reynolds stress and viscous terms are symmetric about the equator, but Maxwell stress term is very asymmetric, as expected.

For this hemispherical dynamo, the asymmetric Maxwell stress maintains the asymmderic meridional circulation. A small asymmetric perturbation in the magnetic field will grow.

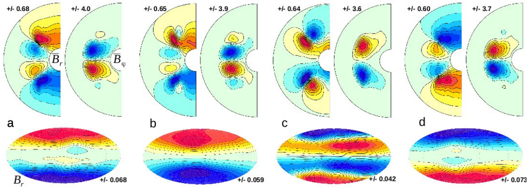
Trigram plot



D/Q refer to the magnetic energy in the dipole and quadrupole contributions, NA to the energy in the non-axisymmetric components. An almost purely dipolar dynamo, like Saturn, sits near point D. The green square dynamo is mostly quadrupolar, with a small non-axisymmetric component. Uranus and Neptune are primarily non-axisymmetric.



With a Saturn-like conductivity profile, the dipolar solutions are oscillatory. The period corrresponds to many thousands of years, so this is consistent with observations.



Meridional sections of zonally averaged radial and azimuthal field, and surface radial field at four times in the cycle.

There are times when the field is somewhat Saturn-like, but the non-axisymmetric components are always larger than Saturn's.

Conclusions

• Remarkable diversity of magnetic field morphology. Large band of r_d values between Jupiter and Saturn, which are quadrupolar rather than dipolar. Both Jupiter and Saturn lie just outside the quadrupolar belt.

• Saturn-like fields possible, but tilt angle of field typically $1^\circ \sim 2^\circ$, whereas Saturn's tilt is certainly less than 0.06°. A stably stratified layer is necessary to model Saturn's very axisymmetric field.

• Broader equatorial jet found in Saturn in agreement with observation. Our jets are slightly broader than those in the giant planets: better modelling of small scale turbulence needed?

• $r_d \sim 0.65$ dipolar dynamos are oscillatory reversing dynamos with long periods. Shear gets inside the mTC creating dynamo waves.

• Helicity reversal occurs in the metallic regions of the anelastic models. May be connected with Maxwell stresses and the consequent meridional circulation.