Axisymmetric steady solutions in an idealized model of atmospheric general circulations: Hadley circulation and super-rotation

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SUMMARY: We explored steady axisymmetric (2D) solutions of primitive equations of the Boussinesq fluid in a very wide parameter range including both Held and Hou (1980) on the study of the Hadley circulation and Matsuda (1980, 1982) on the study of the super-rotation. We estimated the values of parameters when the “transition” of the circulation type occurs. Furthermore, non-axisymmetric (3D) solutions were calculated and were compared with 2D solutions. In 3D solutions, when the planet rotation is slow, there is angular momentum transport to low latitudes as Matsuda assumed, but its meridional distribution is not as simple as a horizontal eddy diffusion in the 2D model.

1. Introduction

Held and Hou (1980, HH80 hereafter) studied the dynamics of the Hadley circulation of the Earth by using an idealized axisymmetric 2D model with no horizontal eddy diffusion, which is known as the Held-Hou model. On the other hand, the mechanism of the super-rotation of the Venus was studied by Gierasch (1975). The essence of the Gierasch mechanism is the mean meridional circulation under the large horizontal eddy diffusion. The Gierasch mechanism was studied by Matsuda (1980, 1982, M80/82 hereafter) with a Boussinesq fluid model.

Actually, both HH80 and M80/82 used the same system:

- *the primitive equations of the Boussinesq fluid with a Newtonian heating/cooling, assuming a steady state, and axial and equatorial symmetries.*

The main differences between them are the values of the horizontal eddy diffusion coefficient (ν_H) and the angular velocity of the planet (Ω), as follows:

	Held and Hou (1980)	Matsuda (1980/1982)
ν_H	ZERO	VERY LARGE
Ω	FAST like the EARTH	SLOW like the VENUS

We explored steady 2D solutions in a very wide parameter range including both HH80 and M80/82. We also calculated 3D solutions in the same range, and compared them with 2D solutions.

2. Description of the System

The axisymmetric governing equations are:

Momentum equations

$$\frac{v}{a}\frac{\partial u}{\partial\phi} + w\frac{\partial u}{\partial z} - \frac{uv\tan\phi}{a} - 2\Omega v\sin\phi = \nu_H D_H(u) + \nu_V\frac{\partial^2 u}{\partial z^2},$$

$$\frac{v}{a}\frac{\partial v}{\partial\phi} + w\frac{\partial v}{\partial z} + \frac{u^2\tan\phi}{a} + 2\Omega u\sin\phi = -\frac{1}{a}\frac{\partial\Phi}{\partial\phi} + \nu_H D_H(v) + \nu_V\frac{\partial^2 v}{\partial z^2},$$

Thermodynamic equation

$$\frac{v}{a}\frac{\partial\Theta}{\partial\phi} + w\frac{\partial\Theta}{\partial z} = -\frac{\Theta-\Theta_e}{\tau} + \kappa_V\frac{\partial^2\Theta}{\partial z^2},$$

Hydrostatic equation

$$\frac{\partial\Phi}{\partial z} = g\alpha\Theta,$$

Continuity equation

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\cos\phi) + \frac{\partial w}{\partial z} = 0.$$

Potential temperature in radiative equilibrium

$$\frac{\Theta_e}{\Theta_0} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi) + \Delta_V\left(\frac{z}{H} - \frac{1}{2}\right),$$

Horizontal diffusion terms (Becker, 2001)

$$D_H(u) = \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial u}{\partial\phi}\right) - \frac{u}{a^2\cos^2\phi} + \frac{2u}{a^2},$$

$$D_H(v) = \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial v}{\partial\phi}\right) - \frac{v}{a^2\cos^2\phi} + \frac{1}{a}\frac{\partial}{\partial\phi}\left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\cos\phi)\right] + \frac{2v}{a^2}.$$

Boundary conditions

$$w = \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial\Theta}{\partial z} = 0 \quad \text{at} \quad z = H,$$

$$w = \frac{\partial\Theta}{\partial z} = 0,\ \nu_V\frac{\partial u}{\partial z} = Cu,\ \nu_V\frac{\partial v}{\partial z} = Cv \quad \text{at} \quad z = 0,$$

ν_V : vertical momentum diffusion coefficient, κ_V : vertical thermal diffusion coefficient,
τ : time constant for Newtonian heating/cooling, C : drag coefficient,
Δ_H, Δ_V : fractional change of Θ_e from equator to pole, top to bottom,
Θ_0 : global mean of Θ_e, $\alpha = 1/\Theta_0$.

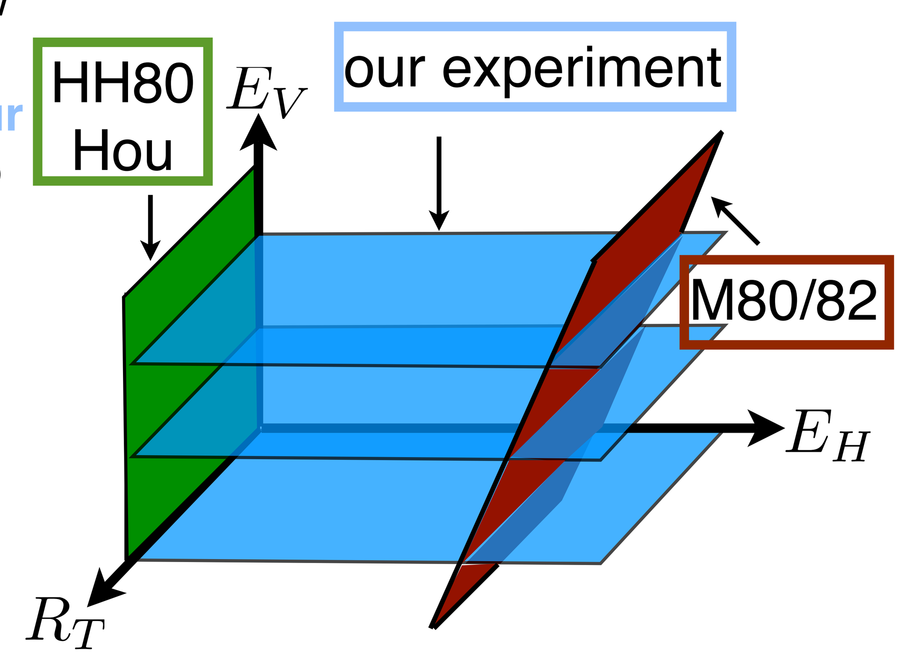
3. Parameter Sweep Experiments

We choose three non-dimensional numbers (R_T, E_H, E_V) for sweep parameters; they are defined as

$$R_T \equiv \frac{gH\Delta_H}{a^2\Omega^2}, \quad E_H \equiv \frac{\nu_H}{a^2\Omega}, \quad E_V \equiv \frac{\nu_V}{H^2\Omega}.$$

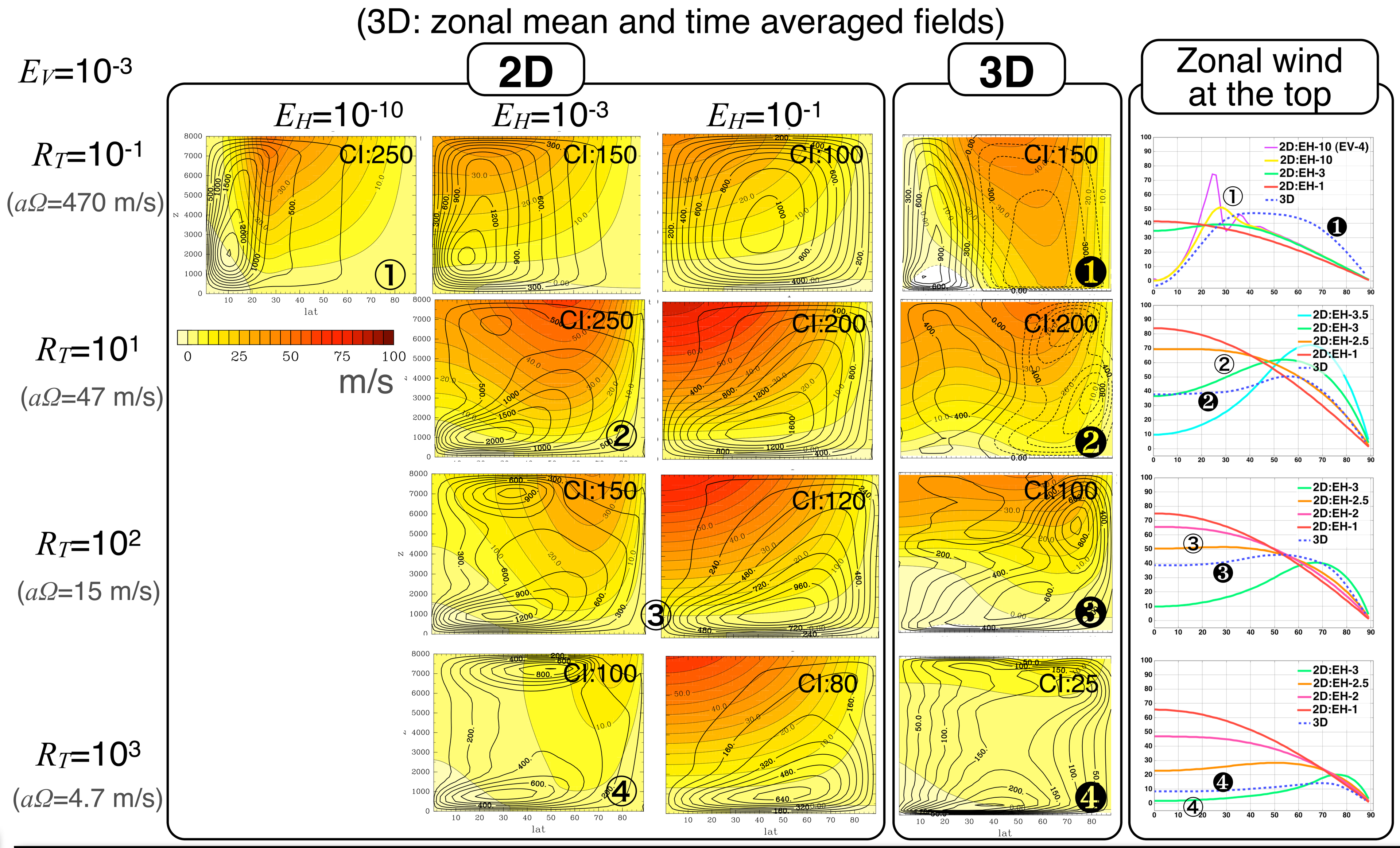
In a parameter space (R_T, E_H, E_V), we can draw the planes which correspond to the parameter range of **HH80 and Hou (1984)**, **M80/82**, and our experiment. Hou applied the Held-Hou model to a slowly rotating planet.

Note that we choose R_T only for a sweep parameter of 3D calculations, because eddy diffusion terms ($\propto E_H, E_V$) in the 2D model represent effects of non-axisymmetric large eddies in the 3D model.



4. Numerical Results

Zonal wind (tone) and Meridional streamfunctions (contour)
(3D: zonal mean and time averaged fields)



5. Dynamical Analysis of 2D Solutions

S : super-rotation intensity

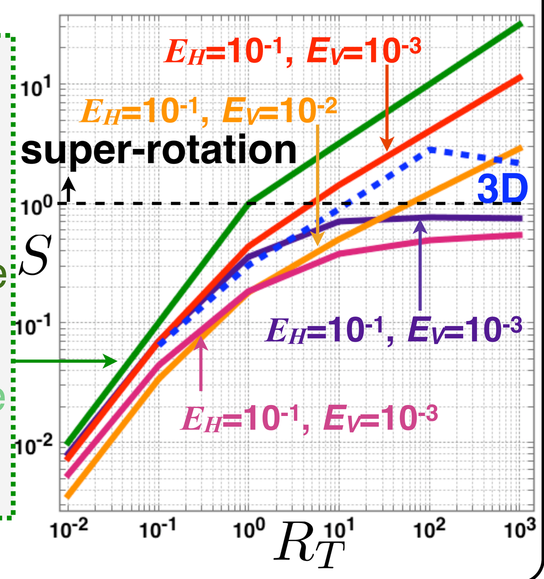
$S \equiv \frac{U}{a\Omega}$: meridional mean zonal wind at the top / rotation speed of the planet

Estimate

$S^2u^2\tan\phi + Su\sin\phi \sim R_T\frac{\partial\Phi}{\partial\phi}$

Geostrophic balance $S \sim R_T (R_T \ll 1)$

Cyclostrophic balance $S \sim \sqrt{R_T} (R_T \gg 1)$

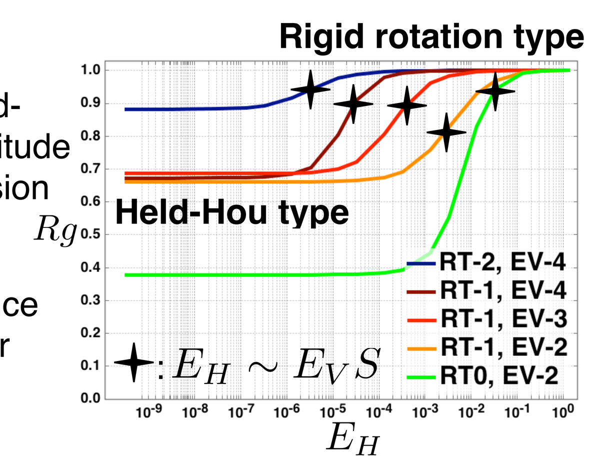


R_g: measure of rigid rotation

R_g : rigid rotation component of zonal KE at the top / zonal KE at the top

Estimate

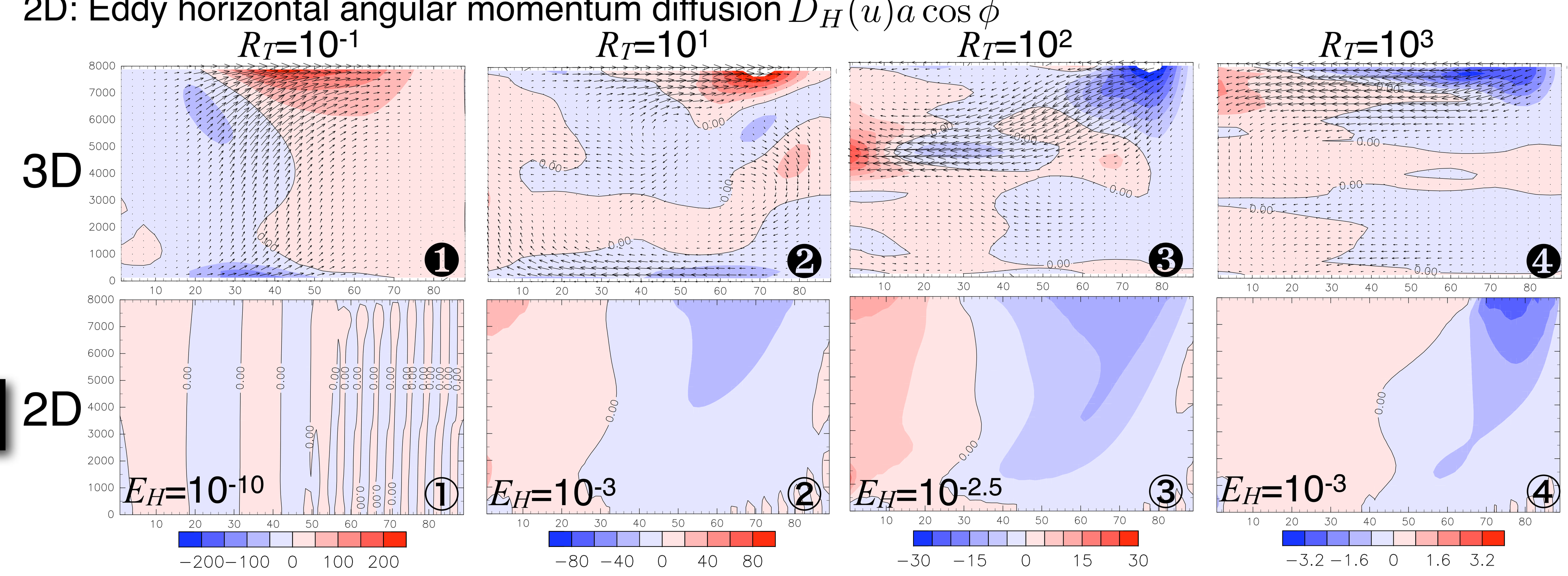
From the zonal wind distribution of the Held-Hou theory, the magnitude of the horizontal diffusion is estimated as $\nu_H a\Omega/a^2$. Considering the balance of the absoute angular momentum equation, “Transition” of the circulation type will occur when $\nu_H\frac{a\Omega}{a^2} \sim \nu_V\frac{U}{H^2} \rightarrow E_H \sim E_V S$



6. Comparison of the Up-gardient Angular Momentum Transport in 2D and 3D

3D: Eddy angular momentum flux $(\overline{M'v'}, \overline{M'w'})$ and its convergence $-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{M'v'}\cos\phi) - \frac{\partial}{\partial\phi}(\overline{M'w'})$

2D: Eddy horizontal angular momentum diffusion $D_H(u)a\cos\phi$



Distributions of the terms which can transport an angular momentum up-gradient are shown. The cases of 2D which have the nearest zonal wind distribution to the 3D solutions are shown here. In the 3D solutions, there is momentum transport to low latitudes when R_T is large (the rotation is slow). However, its distribution is not as simple as 2D horizontal diffusion. This momentum transport is caused by the BAROTROPIC INSTABILITY. When R_T is small, in mid-latitudes, there is momentum transport which are caused by the baroclinic instability, but they are not represented in the 2D model.

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