数値実験：

Mar. 13 '91

回転球面上の2次元乱流

▨ 基礎方程式

$$\frac{\partial \omega^*}{\partial t^*} + \frac{1}{a^{*2}} J(\psi^*, \omega^*) + \frac{2\Omega^*}{a^{*2}} \frac{\partial \psi^*}{\partial \lambda} = \nu_2^* \Delta^* \omega^* - \nu_4^* \Delta^{*2} \omega^* \qquad (1)$$

λ：経度

μ：サイン緯度（$\mu = \sin\phi$，ϕ：緯度）

t^*：時刻

$\psi^*(\lambda, \mu, t^*)$：流れ関数

$$u^* = -\frac{\sqrt{1-\mu^2}}{a^*} \frac{\partial \psi^*}{\partial \mu}$$

$$v^* = \frac{1}{a^* \sqrt{1-\mu^2}} \frac{\partial \psi^*}{\partial \lambda}$$

$\omega^*(\lambda, \mu, t^*)$：渦度（r-成分）

$$\omega^* = \Delta^* \psi^*$$

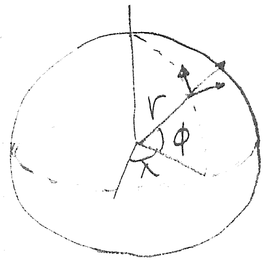
$$J(\alpha, \beta) = \frac{\partial \alpha}{\partial \lambda}\frac{\partial \beta}{\partial \mu} - \frac{\partial \alpha}{\partial \mu}\frac{\partial \beta}{\partial \lambda}$$

$$\Delta^* = \frac{1}{a^{*2}} \left[\frac{1}{1-\mu^2} \frac{\partial^2}{\partial \lambda^2} + \frac{\partial}{\partial \mu} \left\{ (1-\mu^2) \frac{\partial}{\partial \mu} \right\} \right]$$

a^*：惑星半径

Ω^*：惑星自転角速度

球座標 (λ, ϕ, r)



$$\begin{cases} \xi_1 = \lambda \\ \xi_2 = \phi \\ \xi_3 = r \end{cases} \qquad \begin{cases} h_1 = r\cos\phi \\ h_2 = r \\ h_3 = 1 \end{cases}$$

	e_λ	e_ϕ	e_r
$\frac{\partial}{\partial\lambda}$	$e_\phi \sin\phi - e_r \cos\phi$	$-e_\lambda \sin\phi$	$e_\lambda \cos\phi$
$\frac{\partial}{\partial\phi}$	0	$-e_r$	e_ϕ
$\frac{\partial}{\partial r}$	0	0	0

$$\nabla = e_\lambda \frac{1}{r\cos\phi}\frac{\partial}{\partial\lambda} + e_\phi \frac{1}{r}\frac{\partial}{\partial\phi} + e_r \frac{\partial}{\partial r}$$

$$\mathrm{grad}\,\Phi = e_\lambda \frac{1}{r\cos\phi}\frac{\partial\Phi}{\partial\lambda} + e_\phi \frac{1}{r}\frac{\partial\Phi}{\partial\phi} + e_r \frac{\partial\Phi}{\partial r}$$

$$\mathrm{div}\,A = \frac{1}{r\cos\phi}\frac{\partial A_\lambda}{\partial\lambda} + \frac{1}{r\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi A_\lambda) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r$$

$$\mathrm{rot}\,A = e_\lambda \frac{1}{r}\left\{\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(r A_\phi)\right\}$$
$$+ e_\phi \frac{1}{r}\left\{\frac{\partial}{\partial r}(r A_\lambda) - \frac{1}{\cos\phi}\frac{\partial A_r}{\partial\lambda}\right\}$$
$$+ e_r \frac{1}{r\cos\phi}\left\{\frac{\partial A_\phi}{\partial\lambda} - \frac{\partial}{\partial\phi}(\cos\phi A_\lambda)\right\}$$

$$\triangle\Phi = \frac{1}{r^2\cos^2\phi}\frac{\partial^2\Phi}{\partial\lambda^2} + \frac{1}{r^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\Phi}{\partial\phi}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right.$$

$a\cos\phi$

$\partial\phi$ / a

球面での積分

$$\int dS = \int_0^{2\pi} d\lambda \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \; a^2 \cos\phi$$

$$= \int_0^{2\pi} d\lambda \int_{-1}^{1} d\mu \quad a^2$$

部分積分　（A, B は球面上の smooth な関数）

- $$\int A \Delta B \, dS = -\int \mathrm{grad}\, A \cdot \mathrm{grad}\, B \, dS$$
$$= \int (\Delta A) B \, dS$$

- $$\int A \frac{\partial B}{\partial \lambda} dS = -\int \frac{\partial A}{\partial \lambda} B \, dS$$

Jacobian の積分

$$\int \frac{1}{\cos\phi} \frac{\partial(A, B)}{\partial(\lambda, \phi)} dS = \int \frac{\partial(A, B)}{\partial(\lambda, \mu)} dS$$
$$= 0$$

$$\mathrm{div}\,\mathbb{A} = \frac{1}{r^2\cos\phi}\left\{\frac{\partial}{\partial\lambda}(rA_\lambda) + \frac{\partial}{\partial\phi}(r\cos\phi A_\phi) + \frac{\partial}{\partial r}(r^2\cos\phi A_r)\right\}$$

$$= \frac{1}{r\cos\phi}\frac{\partial A_\lambda}{\partial\lambda} + \frac{1}{r\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi A_\phi) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r)$$

//

$$\mathrm{rot}\,\mathbb{A} = \frac{1}{r^2\cos\phi}\begin{vmatrix} r\cos\phi\, e_\lambda & r e_\phi & e_r \\ \frac{\partial}{\partial\lambda} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial r} \\ r\cos\phi A_\lambda & rA_\phi & A_r \end{vmatrix}$$

$$= e_\lambda \frac{r\cos\phi}{r^2\cos\phi}\left\{\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right\}$$
$$+ e_\phi \frac{r}{r^2\cos\phi}\left\{\frac{\partial}{\partial r}(r\cos\phi A_\lambda) - \frac{\partial A_r}{\partial\lambda}\right\}$$
$$+ e_r \frac{1}{r^2\cos\phi}\left\{\frac{\partial}{\partial\lambda}(rA_\phi) - \frac{\partial}{\partial\phi}(r\cos\phi A_\lambda)\right\}$$

$$= e_\lambda \frac{1}{r}\left\{\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right\}$$
$$+ e_\phi \frac{1}{r}\left\{\frac{\partial}{\partial r}(rA_\lambda) - \frac{1}{\cos\phi}\frac{\partial A_r}{\partial\lambda}\right\}$$
$$+ e_r \frac{1}{r\cos\phi}\left\{\frac{\partial A_\phi}{\partial\lambda} - \frac{\partial}{\partial\phi}(\cos\phi A_\lambda)\right\}$$

//

$$\Delta\Phi = \nabla\cdot\nabla\Phi$$
$$= \frac{1}{r^2\cos\phi}\left\{\frac{\partial}{\partial\lambda}\left(\frac{r}{r\cos\phi}\frac{\partial\Phi}{\partial\lambda}\right) + \frac{\partial}{\partial\phi}\left(\frac{r\cos\phi}{r}\frac{\partial\Phi}{\partial\phi}\right) + \frac{\partial}{\partial r}\left(\frac{r^2\cos\phi}{1}\frac{\partial\Phi}{\partial r}\right)\right\}$$

$$= \frac{1}{r^2\cos^2\phi}\frac{\partial^2\Phi}{\partial\lambda^2} + \frac{1}{r^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\Phi}{\partial\phi}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)$$

//

ν_2^*, ν_4^* : 粘性（超粘性）係数

$$\boldsymbol{u}^* = \boldsymbol{e}_r \times \mathrm{grad}\,\psi^*$$

$$= \begin{vmatrix} \boldsymbol{e}_\lambda & \boldsymbol{e}_\phi & \boldsymbol{e}_r \\ 0 & 0 & 1 \\ \frac{1}{a^*\cos\phi}\frac{\partial \psi^*}{\partial \lambda} & \frac{1}{a^*}\frac{\partial \psi^*}{\partial \phi} & \frac{\partial \psi^*}{\partial r} \end{vmatrix}$$

$\frac{d\mu}{d\phi} = \frac{\partial}{\partial\phi}\sin\phi = \cos\phi$

$$= -\frac{1}{a^*}\frac{\partial \psi^*}{\partial \phi}\boldsymbol{e}_\lambda + \frac{1}{a^*\cos\phi}\frac{\partial \psi^*}{\partial \lambda}\boldsymbol{e}_\phi$$

$$= \underbrace{-\frac{\sqrt{1-\mu^2}}{a^*}\frac{\partial \psi^*}{\partial \mu}}_{u^*}\boldsymbol{e}_\lambda + \underbrace{\frac{1}{a^*\sqrt{1-\mu^2}}\frac{\partial \psi^*}{\partial \lambda}}_{v^*}\boldsymbol{e}_\phi \quad //$$

$$\omega^* = \boldsymbol{e}_r \cdot \mathrm{rot}\,\boldsymbol{u}$$

$$= \frac{1}{a^*\cos\phi}\left\{\frac{\partial v^*}{\partial \lambda} - \frac{\partial}{\partial \phi}(\cos\phi\, u^*)\right\}$$

$$= \frac{1}{a^*\sqrt{1-\mu^2}}\left\{\frac{1}{a^*\sqrt{1-\mu^2}}\frac{\partial^2 \psi^*}{\partial \lambda^2} + \sqrt{1-\mu^2}\frac{\partial}{\partial \mu}\left(\frac{1-\mu^2}{a^*}\frac{\partial \psi^*}{\partial \mu}\right)\right\}$$

$$= \frac{1}{a^{*2}}\left[\frac{1}{1-\mu^2}\frac{\partial^2 \psi^*}{\partial \lambda^2} + \frac{\partial}{\partial \mu}\left\{(1-\mu^2)\frac{\partial \psi^*}{\partial \mu}\right\}\right]$$

$$= \Delta^*\psi^* \quad //$$

(1) →

$$\frac{\partial}{\partial t^*}\Delta^*\psi^* + \frac{1}{a^{*2}}J(\psi^*, \Delta^*\psi^*) + \frac{2\Omega^*}{a^{*2}}\frac{\partial \psi^*}{\partial \lambda}$$

$$= \nu_2^*\Delta^{*2}\psi^* - \nu_4\Delta^{*3}\psi^* \qquad (2)$$

▨ 無次元化.

$$\begin{cases} t^* = Tt, & t:\text{無次元時刻} \\ \psi^* = \dfrac{L^2}{T}\psi, & \psi:\text{無次元流れ関数} \\ \Delta^* = \dfrac{1}{a^{*2}}\Delta, & \Delta:\text{無次元ラプラシアン} \end{cases}$$

T : 時間の尺度

L : 距離の尺度

$$\Delta = \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\lambda^2} + \frac{\partial}{\partial\mu}\left\{(1-\mu^2)\frac{\partial}{\partial\mu}\right\}$$

$$\omega^* = \Delta^*\psi^* = \frac{1}{a^{*2}}\cdot\frac{L^2}{T}\Delta\psi$$

(2) →

$$\frac{L^2}{a^{*2}T^2}\frac{\partial}{\partial t}\Delta\psi + \frac{L^4}{a^{*4}T^2}J(\psi, \Delta\psi) + \frac{2L^2\Omega^*}{a^{*2}T}\frac{\partial\psi}{\partial\lambda}$$

$$= \frac{\nu_2^* L^2}{a^{*4}T}\Delta^2\psi - \frac{\nu_4^* L^2}{a^{*6}T}\Delta^3\psi$$

$$\therefore \frac{\partial}{\partial t}\Delta\psi + \frac{L^2}{a^{*2}}J(\psi, \Delta\psi) + 2\Omega^* T\frac{\partial\psi}{\partial\lambda}$$

$$= \frac{\nu_2^* T}{a^{*2}}\Delta^2\psi - \frac{\nu_4^* T}{a^{*4}}\Delta^3\psi$$

$$\frac{\partial}{\partial t}\Delta\psi + r^2 J(\psi, \Delta\psi) + 2\Omega\frac{\partial\psi}{\partial\lambda} = \nu_2 \Delta^2\psi - \nu_4 \Delta^3\psi \quad (3)$$

$\Omega^* = \frac{1}{T}\Omega$　　Ω : 無次元 自転角速度

$\nu_2^* = \frac{a^{*2}}{T}\nu_2 = \frac{a^* U}{L_0}$　　ν_2 : 無次元 粘性係数

$\nu_4^* = \frac{a^{*4}}{T}\nu_4 = \frac{a^{*3} U}{L_0^3}$　　ν_4 : 無次元 超粘性係数

$r^2 = \frac{L^2}{a^{*2}}$　　r : 距離の尺度と惑星半径との比.

$0 < r \lesssim 2\pi$

$L_{max} = 2\pi a^*$

$2\Omega = 2T\Omega^* = \frac{2\Omega^* L}{U} = R_0^{-1}$ (ロスビー数)$^{-1}$

$(U = L/T)$

無次元パラメータ : r^2, 2Ω (R_0^{-1}), ν_2, ν_4 が同一であれば, 運動は力学的に相似になる。

- $r^2 = 1$　$L = a^*$ とする
- 初期値　$\mathcal{E}(0) = 1$, fix　　Ω : 実験パラメータ $0 < \Omega < \infty$
- $\nu_2 = 0$, $\nu_4 = \mathrm{const.}$
- 別の初期値スペクトル形 → 別の ν_4.

L_0が変わると → dissipation wavenumber

▨ 運動エネルギーとエンストロフィー

単位面積あたりの平均運動エネルギー

$$\mathcal{E}^*(t) = \frac{1}{4\pi a^{*2}} \int_0^{2\pi}\int_{-1}^{1} \frac{1}{2}(u^{*2} + v^{*2})\, a^{*2}\, d\mu\, d\lambda$$

$$= \frac{1}{4\pi} \int_0^{2\pi}\int_{-1}^{1} \frac{1}{2}(u^{*2} + v^{*2})\, d\mu\, d\lambda \qquad [m^2/s^2]$$

$$= \frac{1}{4\pi} \int_0^{2\pi}\int_{-1}^{1} \frac{1}{2a^{*2}} \left\{ (1-\mu^2)\left(\frac{\partial \psi^*}{\partial \mu}\right)^2 + \frac{1}{1-\mu^2}\left(\frac{\partial \psi^*}{\partial \lambda}\right)^2 \right\} d\mu\, d\lambda$$

$$= \frac{1}{8\pi a^{*2}} \int_0^{2\pi} \left[\cancel{(1-\mu)^2 \psi^* \frac{\partial \psi^*}{\partial \mu}\Big|_{-1}^{1}}^{\to 0} - \int_{-1}^{1} \psi^* \frac{\partial}{\partial \mu}\left\{(1-\mu^2)\frac{\partial \psi^*}{\partial \mu}\right\} d\mu \right] d\lambda$$
$$+ \frac{1}{8\pi a^{*2}} \int_{-1}^{1} \left[\cancel{\frac{1}{1-\mu^2} \psi^* \frac{\partial \psi^*}{\partial \lambda}\Big|_0^{2\pi}}^{\to 0} - \int_0^{2\pi} \frac{1}{1-\mu^2} \psi^* \frac{\partial^2 \psi^*}{\partial \lambda^2} d\lambda \right] d\mu$$

$$= \frac{-1}{8\pi a^{*2}} \int_0^{2\pi}\int_{-1}^{1} \psi^* \left[\frac{1}{1-\mu^2}\frac{\partial^2 \psi^*}{\partial \lambda^2} + \frac{\partial}{\partial \mu}\left\{(1-\mu^2)\frac{\partial \psi^*}{\partial \mu}\right\} \right] d\mu\, d\lambda$$

$$= \frac{-1}{8\pi} \int_0^{2\pi}\int_{-1}^{1} \psi^* \Delta^* \psi^*\, d\mu\, d\lambda$$

(変数の無次元化)

$$= \frac{-1}{8\pi} \cdot \frac{L^4}{T^2 a^{*2}} \int_0^{2\pi}\int_{-1}^{1} \psi \Delta \psi\, d\mu\, d\lambda$$

$$= \frac{-1}{8\pi} \cdot \boxed{U^2 \gamma^2} \int_0^{2\pi}\int_{-1}^{1} \psi \Delta \psi\, d\mu\, d\lambda$$

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エネルギースペクトル関数

$$\mathcal{E}^*(t) = -\frac{U^2 r^2}{2}\frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\psi_n^m \tilde{Y}_n^m \times \sum_{n'=0}^{\infty}\sum_{m'=-n'}^{n'} -n'(n'+1)\,\psi_{n'}^{m'}\tilde{Y}_{n'}^{m'}\,d\lambda\,d\mu$$

(Note: E-3)

$$= \frac{U^2 r^2}{2}\sum_n\sum_m n(n+1)\,|\psi_n^m(t)|^2$$

エネルギースペクトル関数 : $E^*(n,t)$

$$E^*(n,t) \equiv \frac{U^2 r^2}{2}\sum_{m=-n}^{n} n(n+1)\,|\psi_n^m(t)|^2$$

$$\mathcal{E}^*(t) = \sum_{n=0}^{\infty} E^*(n,t)$$

無次元化

$$\mathcal{E}^*(t) = U^2 r^2\,\mathcal{E}(t)$$

$$E^*(n,t) = U^2 r^2\,E(n,t)$$

但し、

$$\mathcal{E}(t) = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m=-n}^{n} n(n+1)\,|\psi_n^m(t)|^2$$

$$E(n,t) = \frac{1}{2}\sum_{m=-n}^{n} n(n+1)\,|\psi_n^m(t)|^2$$

$$\left(\mathcal{E}(t) = \sum_{n=0}^{\infty} E(n,t)\right)$$

単位面積あたりの平均エンストロフィー

$$Q^*(t) = \frac{1}{4\pi a^{*2}} \int_0^{2\pi}\int_{-1}^{1} \frac{1}{2}(\Delta^* \psi^*)^2 a^{*2} d\mu d\lambda$$

$$= \frac{1}{4\pi} \int_0^{2\pi}\int_{-1}^{1} \frac{1}{2}(\Delta^* \psi^*)^2 d\mu d\lambda \qquad [1/s^2]$$

(変数の無次元化)

$$= \frac{1}{8\pi} \frac{L^4}{a^{*4} T^2} \int_0^{2\pi}\int_{-1}^{1} (\Delta \psi)^2 d\mu d\lambda$$

$$= \frac{1}{8\pi} \frac{1}{T^2} \gamma^4 \int_0^{2\pi}\int_{-1}^{1} (\Delta \psi)^2 d\mu d\lambda$$

エンストロフィースペクトル関数

$$Q^*(t) = \frac{\gamma^4}{2T^2} \sum_n \sum_m n^2(n+1)^2 |\psi_n^m(t)|^2$$

エンストロフィースペクトル関数 : $Q^*(n,t)$

$$Q^*(n,t) \equiv \frac{\gamma^4}{2T^2} \sum_{m=-n}^{n} n^2(n+1)^2 |\psi_n^m(t)|^2$$

$$Q^*(t) = \sum_{n=0}^{N} Q^*(n,t)$$

無次元化

$$Q^*(t) = \frac{\gamma^4}{T^2} Q(t)$$

$$Q^*(n,t) = \frac{\gamma^4}{T^2} Q(n,t)$$

但し,

$$Q(t) = \frac{1}{2} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} n^2(n+1)^2 |\psi_n^m(t)|^2$$

$$Q(n,t) = \frac{1}{2} \sum_{m=-n}^{n} n^2(n+1)^2 |\psi_n^m(t)|^2$$

$$\left(Q(t) = \sum_{n=0}^{\infty} Q(n,t) \right)$$

▨ 初期値.

エネルギースペクトル関数の関数型を固定する。

$$\begin{cases} \mathcal{E}(0) = \sum_{n=2}^{M} E(n,0) = 1 \\ E(n,0) = \alpha n^j e^{-\beta n} \end{cases}$$

← $n=1$：剛体回転成分はゼロとする。

実際には、j, β を与え、$\underline{\mathcal{E}(0) = 1}$ となる α を求める。

GINIT

- j, β を与える
- $\sum_{n=0}^{M} n^j e^{-\beta n}$ の計算
- $\alpha = 1 / \sum_{n=0}^{M} n^j e^{-\beta n}$
- 乱数で n について分配（位相も乱数）→ $\psi_n^m(0)$
- $Q(n,0)$, $Q(0)$
- $U_0 = \sqrt{\mathcal{E}(0)}$

  $L_0 = \sqrt{\mathcal{E}(0) / Q(0)}$

  $T_0 = L_0 / U_0$
- output

（$j=5$, $\beta=0.5$）
↓
$\mathcal{E}(0) = 1$, $Q(0) = 180.$
↓
$U_0 = 1$
$L_0 = 0.0745$
‖
T_0

▨ 初期值：空间对称性

$$
\begin{cases}
\psi^{I}(\lambda,\mu,0) = \psi(\lambda,\mu) = -\psi^{V}(\lambda,\mu,0) \\
\psi^{II}(\lambda,\mu,0) = \psi(-\lambda,\mu) = -\psi^{VI}(\lambda,\mu,0) \\
\psi^{III}(\lambda,\mu,0) = \psi(\lambda,-\mu) = -\psi^{VII}(\lambda,\mu,0) \\
\psi^{IV}(\lambda,\mu,0) = \psi(-\lambda,-\mu) = -\psi^{VIII}(\lambda,\mu,0)
\end{cases}
$$

$2^3 = 8$ cases.

II · VI :

$$
\begin{aligned}
\psi^{II} &= \psi(-\lambda,\mu) \\
&= \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \psi_n^m \tilde{Y}_n^m(-\lambda,\mu) \\
&= \sum_n\sum_m \psi_n^m \tilde{P}_n^m(\mu) e^{-im\lambda} \\
&= \sum_n\sum_{m'} \psi_n^{-m'} \tilde{P}_n^{-m'}(\mu) e^{im'\lambda} \qquad m' = -m \\
&= \sum_n\sum_{m'} \psi_n^{-m'} (-1)^{m'} \tilde{P}_n^{m'} e^{im'\lambda} \\
& \qquad\qquad m = m' \\
&= \sum_n\sum_m (-1)^m \psi_n^{-m} \tilde{Y}_n^m(\lambda,\mu) \\
&= \sum_n\sum_m (\psi_n^m)^* \tilde{Y}_n^m
\end{aligned}
$$

$$
\therefore \left.
\begin{aligned}
\psi^{II}{}_n^m &= (\psi^{I}{}_n^m)^* \\
\psi^{VI}{}_n^m &= -(\psi^{I}{}_n^m)^*
\end{aligned}
\right\}
$$

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III · VII

$$\psi^{III} = \psi(\lambda, -\mu)$$

$$= \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \psi_n^m \tilde{Y}_n^m(\lambda, -\mu)$$

$$= \sum_n \sum_m \psi_n^m (-1)^{n+m} \tilde{Y}_n^m(\lambda, \mu)$$

$$\therefore \quad \left. \begin{aligned} \psi^{III}{}_n^m &= (-1)^{n+m} \psi^{I}{}_n^m \\ \psi^{VII}{}_n^m &= (-1)^{n+m+1} \psi^{I}{}_n^m \end{aligned} \right\}$$

IV · VIII

$$\psi^{IV} = \psi(-\lambda, -\mu)$$

$$= \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \psi_n^m \tilde{Y}_n^m(-\lambda, -\mu)$$

$$= \sum_n \sum_m \psi_n^m (-1)^{n+m} \tilde{Y}_n^m(-\lambda, \mu)$$

$$= \sum_n \sum_m (-1)^{n+m} \psi_n^m \tilde{P}_n^m(\mu) e^{-im\lambda} \qquad m' = -m$$

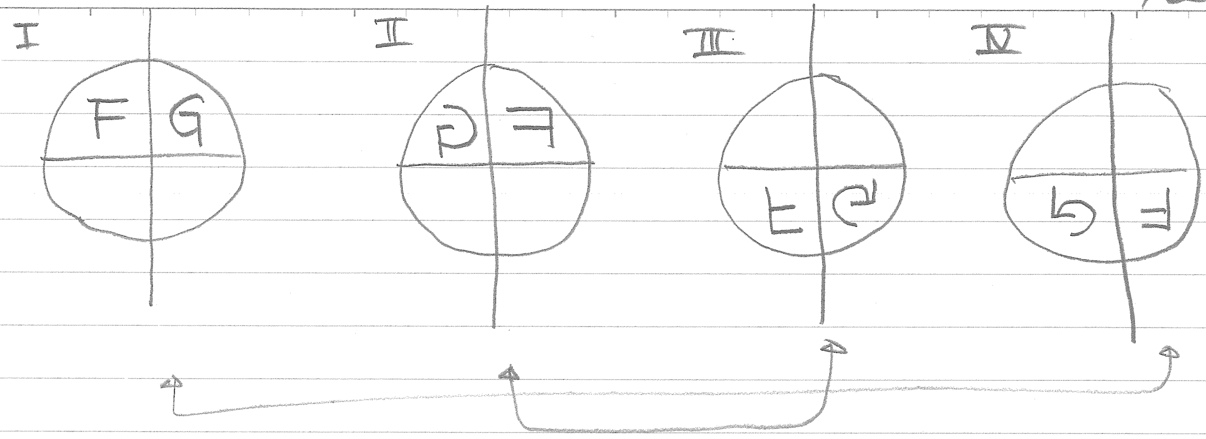
$$= \sum_n \sum_{m'} (-1)^{n-m'} \psi_n^{-m'} (-1)^{m'} \tilde{P}_n^{m'}(\mu) e^{im'\lambda}$$

$$m = m'$$

$$= \sum_n \sum_m (-1)^{n-m} (-1)^m \psi_n^{-m} \tilde{Y}_n^m(\lambda, \mu)$$

$$= \sum_n \sum_m (-1)^{n-m} (\psi_n^m)^* \tilde{Y}_n^m(\lambda, \mu)$$

$$\therefore \quad \left. \begin{aligned} \psi^{IV}{}_n^m &= (-1)^{n-m} (\psi^{I}{}_n^m)^* \\ \psi^{VIII}{}_n^n &= (-1)^{n-m+1} (\psi^{I}{}_n^m)^* \end{aligned} \right\} //$$



但し, $\Omega \to -\Omega$

```
<<< INITIAL STATE >>>
   J =01     A = 0.13022E-03     B = 0.50
   TOTAL ENERGY = 0.10000E+01     TOTAL ENSTROPHY = 0.18001E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.74533E-01

<<< INITIAL STATE >>>
   J =02     A = 0.13022E-03     B = 0.50
   TOTAL ENERGY = 0.10000E+01     TOTAL ENSTROPHY = 0.18001E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.74533E-01
```

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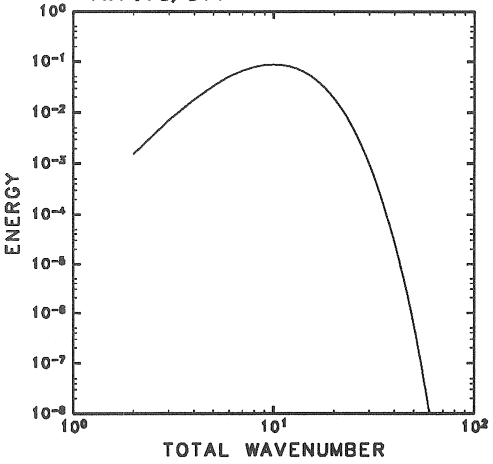

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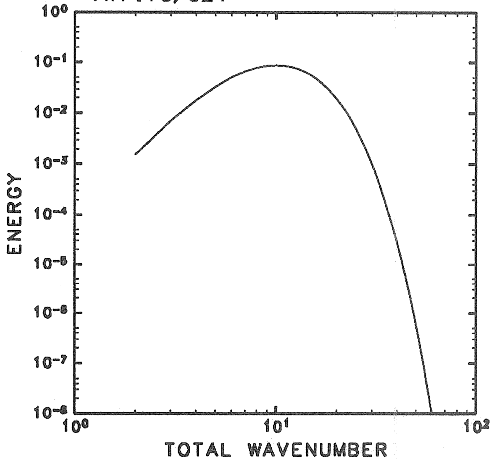

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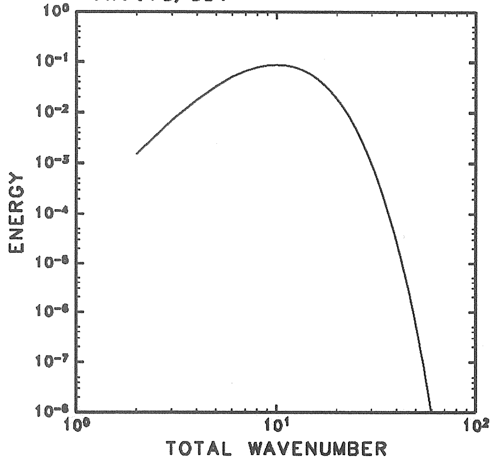

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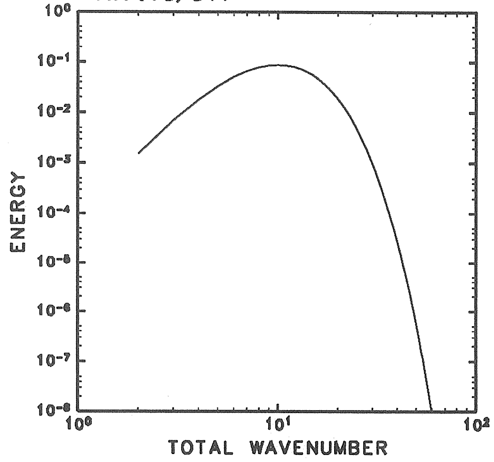

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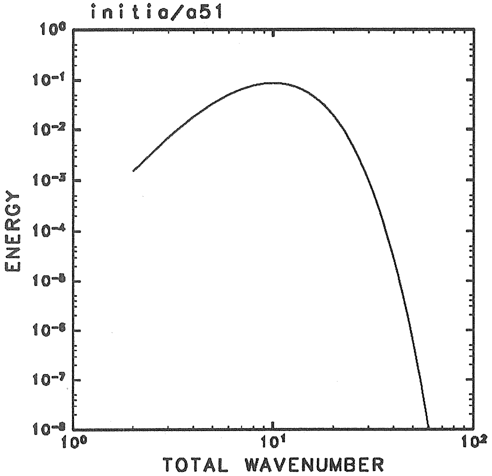

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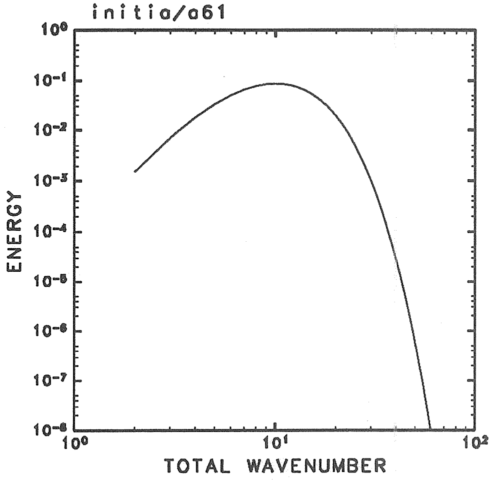

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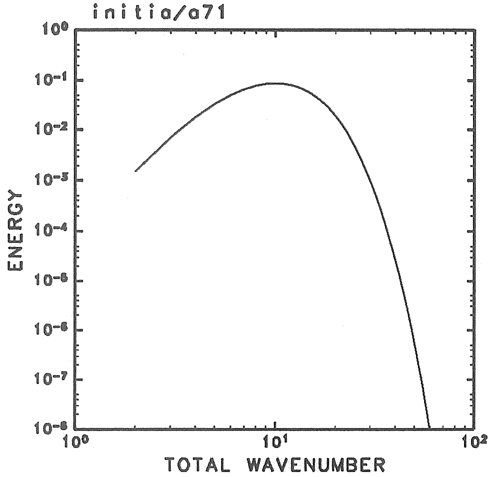

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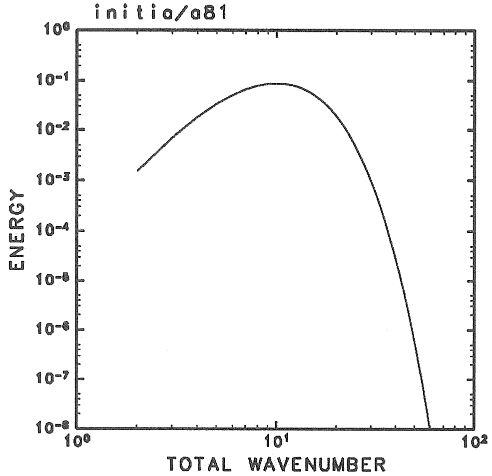

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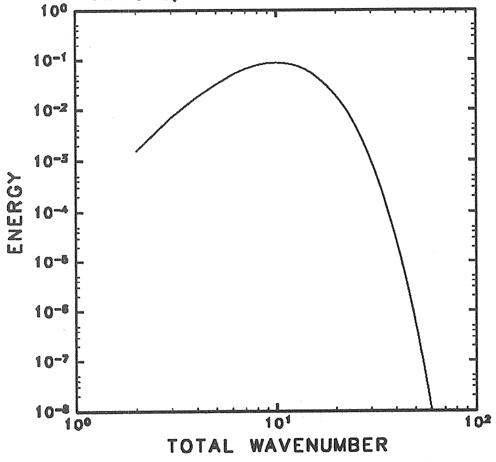

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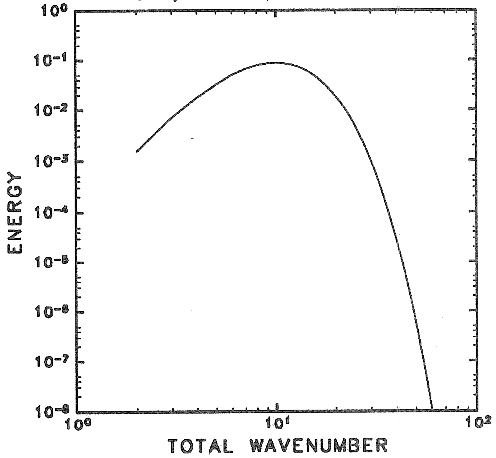

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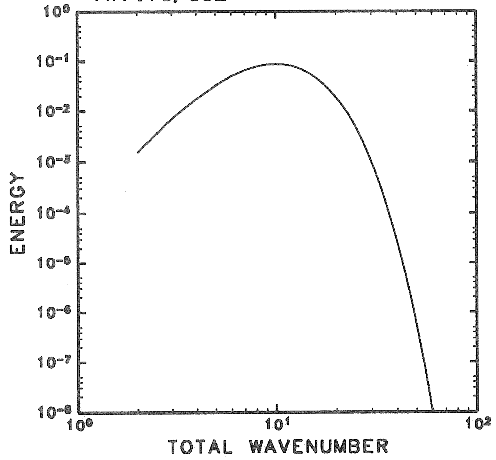

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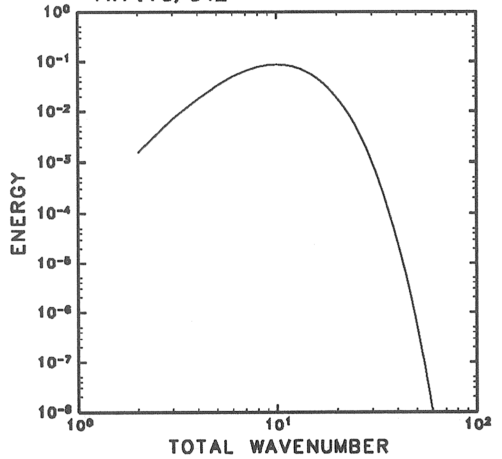

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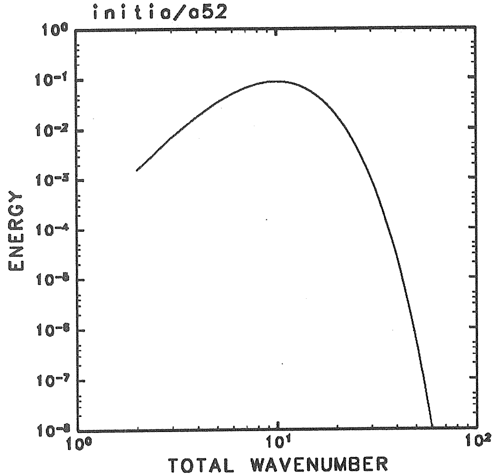

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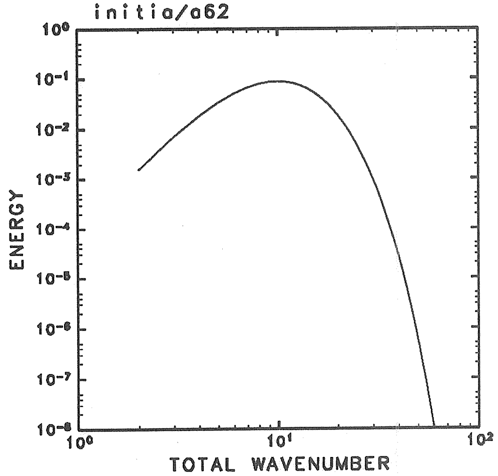

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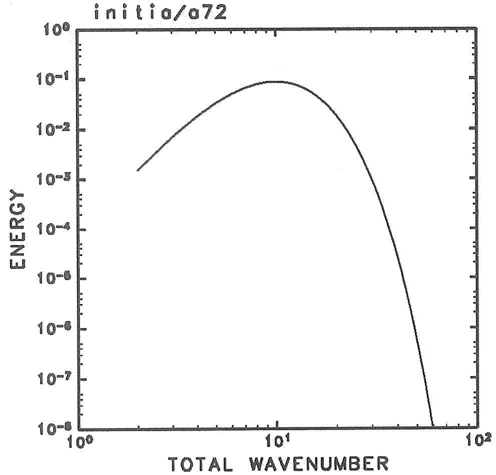

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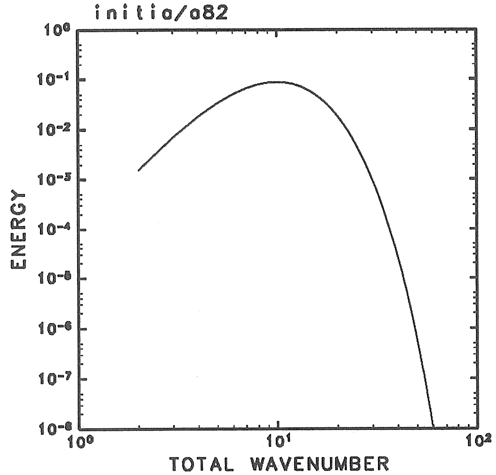
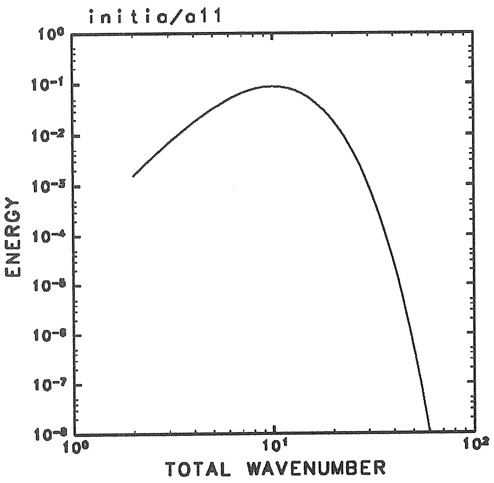
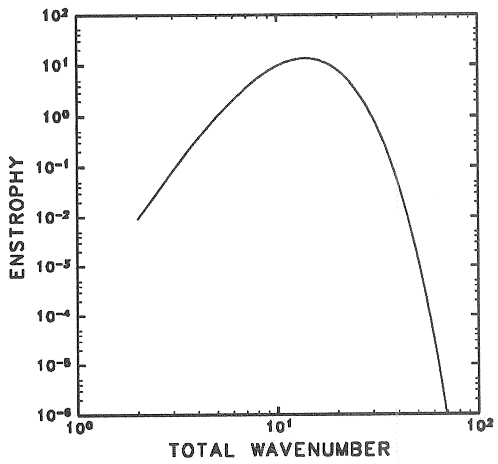
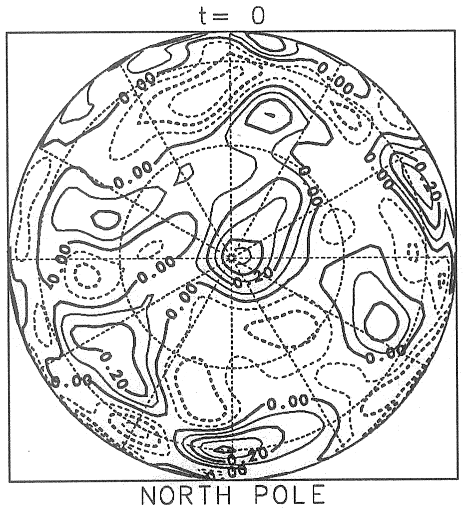



initia/a11
ENERGY
TOTAL WAVENUMBER


ENSTROPHY
TOTAL WAVENUMBER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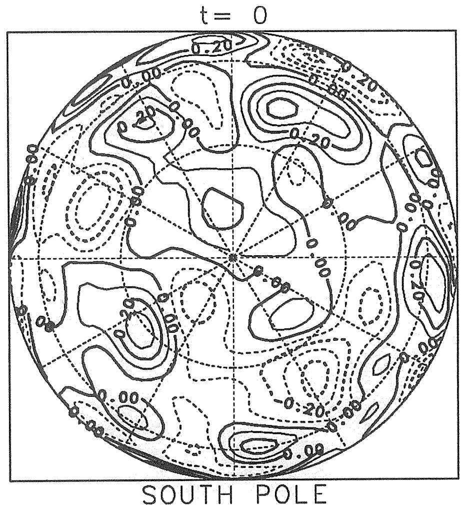
NORTH POLE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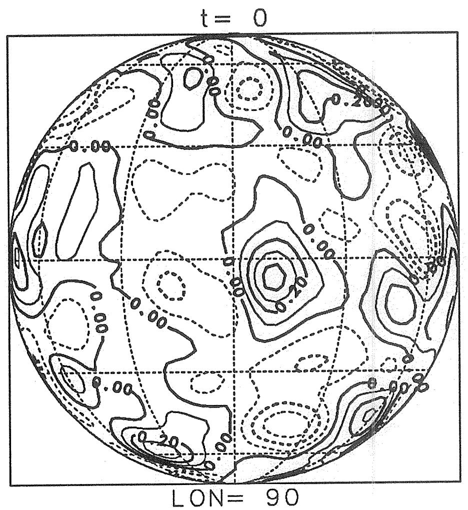
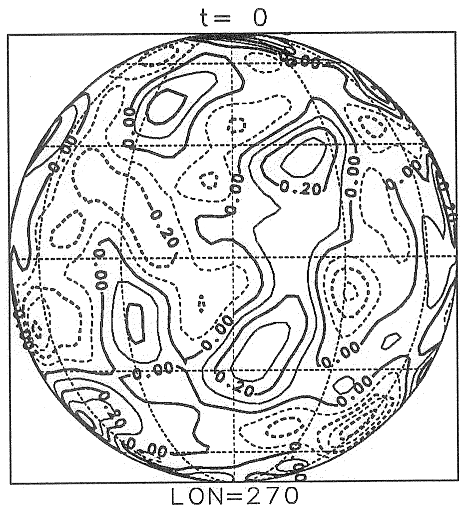
SOUTH POLE


t= 0
LON= 0


t= 0
LON= 90


t= 0
LON=180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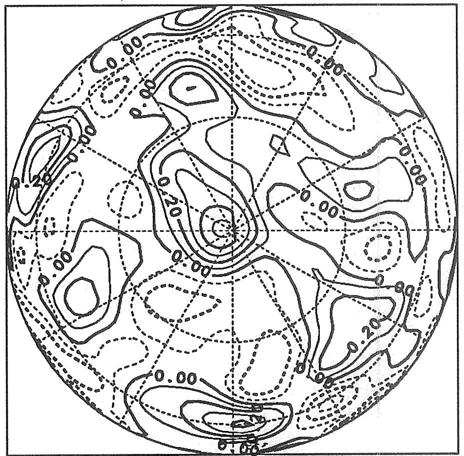
initia/a11

NORTH POLE

initia/a21

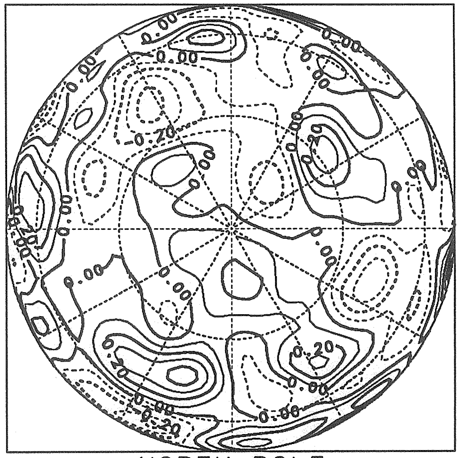


NORTH POLE

initia/a31

NORTH POLE

initia/a41



NORTH POLE

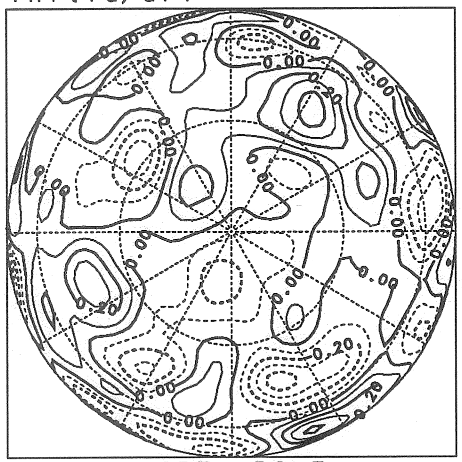
initia/a51

NORTH POLE

initia/a61

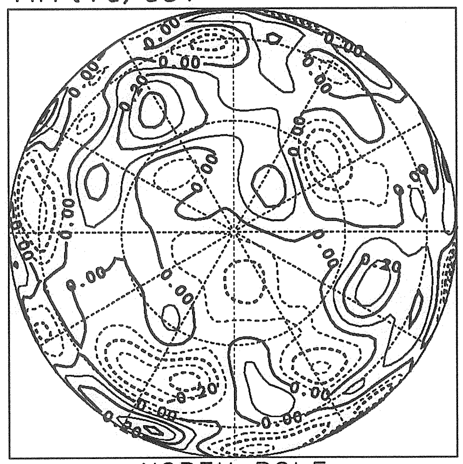
NORTH POLE

initia/a71



NORTH POLE

initia/a81



NORTH POLE

initia/a11

LON= 0

initia/a21

LON= 0

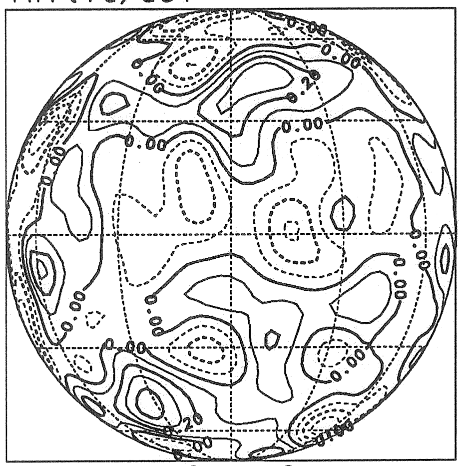
initia/a31

LON= 0

initia/a41

LON= 0

initia/a51



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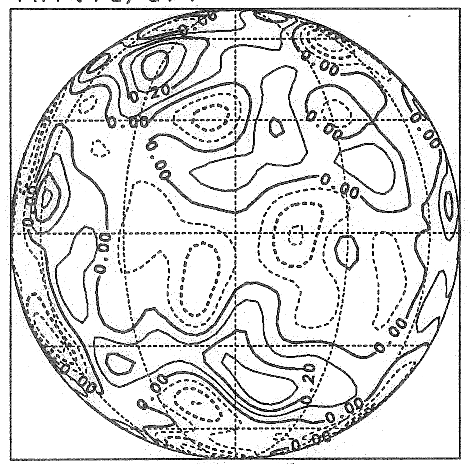
initia/a61

LON= 0

initia/a71



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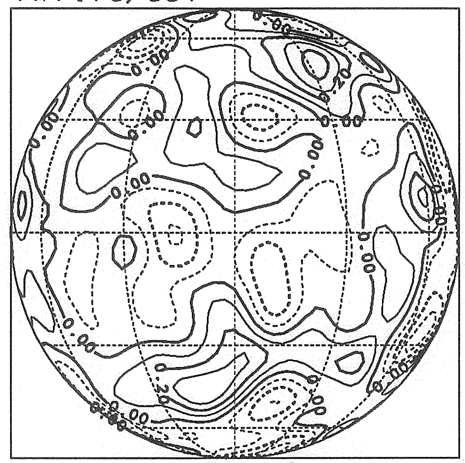
initia/a81



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initia/a12

LON= 0

initia/a22

LON= 0

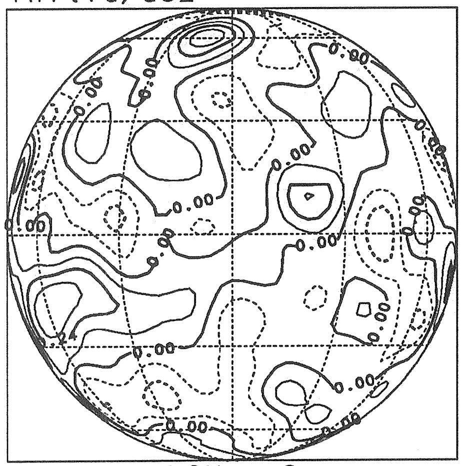
initia/a32

LON= 0

initia/a42

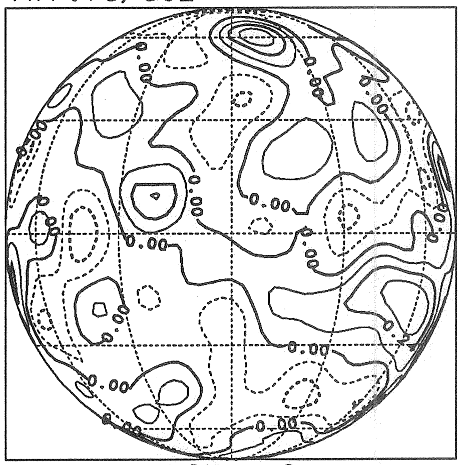
LON= 0

initia/a52



LON= 0

initia/a62



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initia/a72

LON= 0

initia/a82

LON= 0

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AZ001	A21	1.000000	0.898020	0.869120	0.855460	0.845210
AZ001	A31	1.000000	0.898020	0.869120	0.855460	0.845210
AZ001	A41	1.000000	0.899270	0.869710	0.855300	0.846260
AZ001	A51	1.000000	0.898020	0.869120	0.855460	0.845210
AZ001	A61	1.000000	0.899270	0.869710	0.855300	0.846260
AZ001	A71	1.000000	0.899270	0.869710	0.855300	0.846260
AZ001	A81	1.000000	0.898020	0.869120	0.855460	0.845210
AZ002	A11	1.000000	0.901370	0.867110	0.852380	0.844350
AZ002	A21	1.000000	0.898410	0.867120	0.852870	0.845200
AZ002	A31	1.000000	0.900340	0.868870	0.853660	0.845380
AZ002	A41	1.000000	0.901680	0.872670	0.858180	0.848810
AZ002	A51	1.000000	0.900340	0.868870	0.853660	0.845380
AZ002	A61	1.000000	0.901680	0.872670	0.858180	0.848810
AZ002	A71	1.000000	0.901370	0.867110	0.852380	0.844350
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AZ003	A41	1.000000	0.901120	0.874330	0.859310	0.849020
AZ003	A51	1.000000	0.899880	0.869000	0.854590	0.845670
AZ003	A61	1.000000	0.901120	0.874330	0.859310	0.849020
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AZ003	A81	1.000000	0.899040	0.867980	0.851510	0.842060
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AZ005	A31	1.000000	0.902520	0.857910	0.833210	0.816570
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AZ005	A71	1.000000	0.898240	0.857630	0.833970	0.819320
AZ005	A81	1.000000	0.900210	0.860520	0.835750	0.818000
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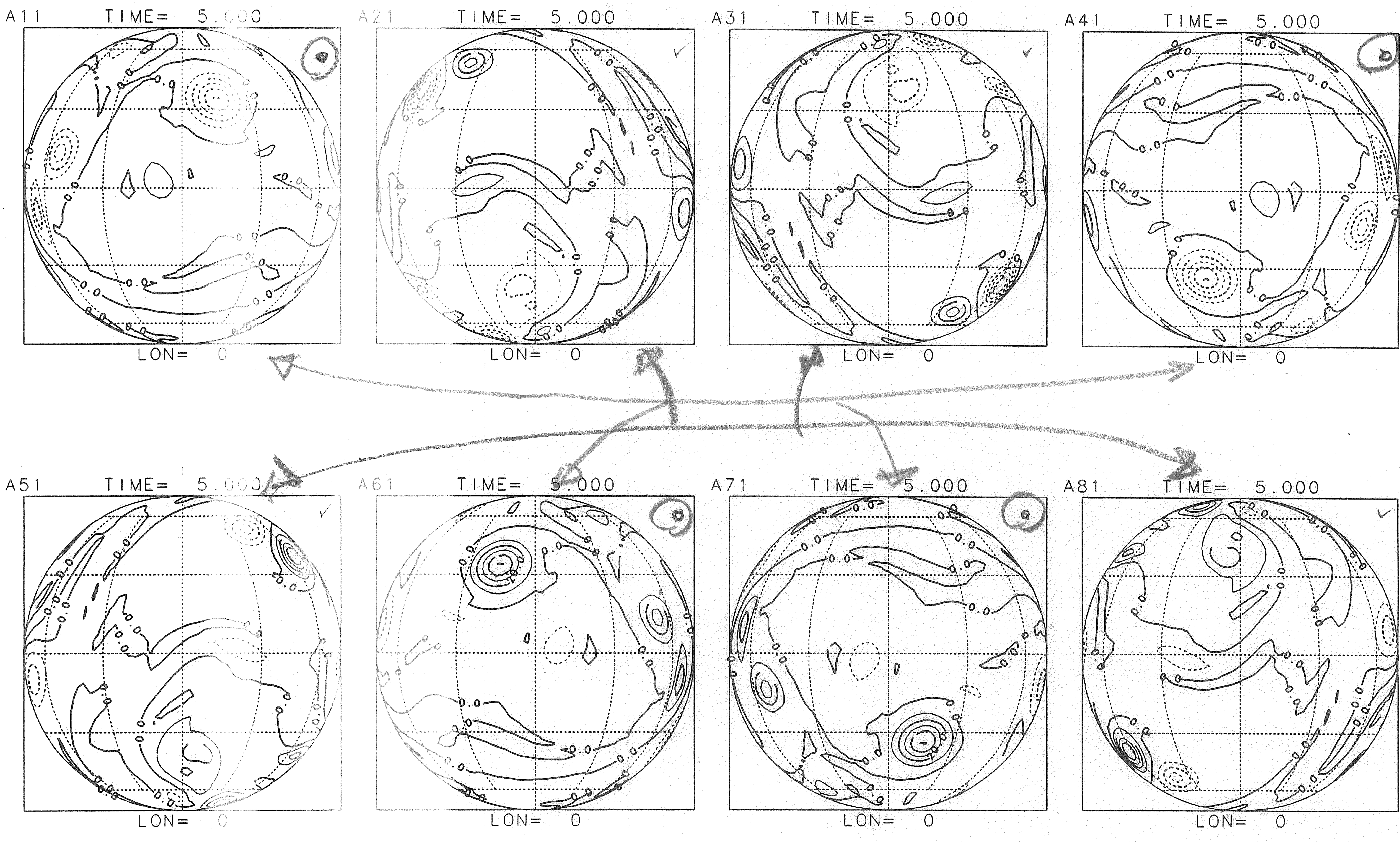
AZ006	A41	1.000000	0.887660	0.830400	0.794730	0.768630
AZ006	A51	1.000000	0.887070	0.832040	0.797280	0.771840
AZ006	A61	1.000000	0.887660	0.830400	0.794730	0.768630
AZ006	A71	1.000000	0.886690	0.830720	0.798710	0.777760
AZ006	A81	1.000000	0.888130	0.834880	0.798510	0.774810

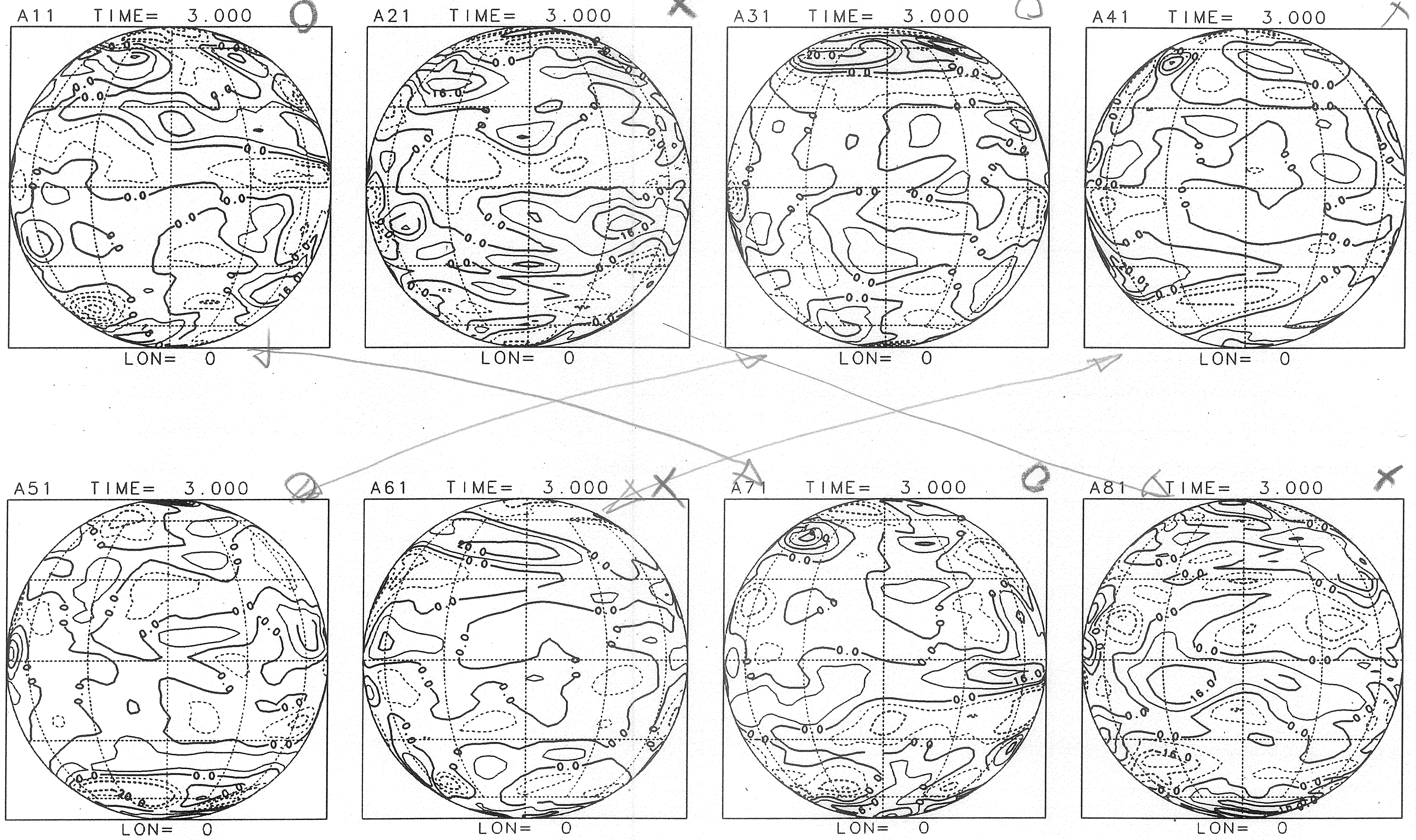
IMAXE



GMAP2.F : PAGE 1 : B52137.A2060.DATA(A11)

$a = 1.0;\ \Omega = 0.0;\ \nu_4 = 1.0 \times 10^{-6}$

91-07-03 16:11:52 S183050U 57971 KEEP



GMAP2 F : PAGE 1 : X57071.AZ100.DATA(A11)

$a= 1.0;\ \Omega=100.0;\ \nu_4= 1.0\mathrm{X}10^{-6}$

▨ 無次元化尺度の与え方
(結果を地球にあてはめて考えるときの尺度)

(仮定)
- i) $r = 1$; $L = a^* = 6.37 \times 10^6 m$
- ii) $\Sigma^*(0) = (10 m/s)^2$

$$U^2 = \frac{1}{r^2} \frac{\Sigma^*(0)}{\Sigma(0)} = (10 m/s)^2$$

$\Sigma(0)$ → 1.0 と決めた.

$$\therefore U = 10 m/s$$

$$T = \frac{L}{U} = \frac{6.37 \times 10^6 m}{10 m/s} = 6.37 \times 10^5 s \doteqdot 176.9 hrs$$

$$\doteqdot 7.37 \text{ days}$$

実験パラメータ: Ω

$$\Omega_e = T\Omega_e^* = 6.37 \times 10^5 s \times 7.292 \times 10^{-5} /s = 46.4$$

e: 地球

$$Ro_e \doteqdot 0.0108$$

$\Omega = 0,\ 25,\ 50,\ 100,\ 200,\ 400$

r: rotation / unit time $= \frac{\Omega}{2\pi} = 0$, 3.98, 7.96, 15.9, 31.8, 63.7

$(Ro) = \infty,\ 0.02,\ 0.01,\ 0.005,$

2π

▨ 初期値の特徴的大きさ

初期値 → $\mathcal{E}(0) = 1.0$

$Q(0) = 180.0$

無次元：

$$U_0 = \sqrt{\mathcal{E}(0)} = 1.0$$

$$L_0 = \sqrt{\mathcal{E}(0)/Q(0)} = 0.0745$$

$$T_0 = L_0 / U_0 = 0.0745$$

次元つき：

$$U_0^* = U_0 U = 10\ m/s$$

$$L_0^* = L_0 L = 0.0745 \times 6.37 \times 10^6\ m$$

$$\fallingdotseq 475\ km$$

$$T_0^* = T_0 T = 0.0745 \times 7.37\ days$$

$$\fallingdotseq 0.549\ days$$

粘性係数

◎ hyper viscosity ν_4 だけ：　$\nu_2 = 0$

（粘性の影響をうける部分をへらす）

◎ 値の決め方

$$\frac{\partial \omega^*}{\partial t^*} \sim \nu_4^* \Delta^{*2} \omega^*$$

$$\frac{1}{T}\frac{\partial}{\partial t} \sim \frac{a^{*4}}{T} \nu_4 \frac{1}{a^{*4}} (\Delta^2)_{max}.$$

$$\nu_4 \sim \frac{1}{M^2 (M+1)^2 \cdot \Delta T}$$

M: truncation total wavenumber

$$\sim \frac{1}{85^2 \cdot 86^2 \cdot 0.0745}$$

$$= 2.51 \times 10^{-7}$$

$$\nu_4 = 1 \times 10^{-6}$$

▨ 時間増分 Δt

○ 格子点法の CFL 条件:

$$|c| \frac{\Delta t^*}{\Delta x^*} \leq 1$$

いま, $|c| \sim U_0^* = U$ 　 $\Delta x^* \sim \frac{2\pi a^*}{NLON}$ と考えるなら
(NLON: transform parameter)

$$\Delta t^* \lesssim \frac{2\pi a^*}{U_0^* NLON}$$

$$\Delta t \leq \frac{2\pi a^*}{U\, NLON} \cdot \frac{1}{T} = \frac{2\pi}{NLON}$$

$$T = \frac{U}{L} a^*$$

$$\fallingdotseq \frac{2\pi}{256} \quad \leftarrow MT=85$$

$$= 0.0245$$

○ 振動型常微分方程式

$$\dot{U} = i\omega^* U$$

matsuno, leapfrog stable for $\omega \Delta t \leq 1$

$$\Delta t \leq \frac{1}{\omega}$$

$$\Delta t = \boxed{5 \times 10^{-3}}$$ 　 $\Delta t^* \sim 53$分

$IMAX = 1000$

$t_{max} = \Delta t \times IMAX = 5$

$t^*_{max} = 5 \times 7.37\,days = 36.85\,days$

▨ 予備実験との対応.

parameters

$a = 1$

$\Omega = 0, 1, \sim 9$ (1 → Ω_e)

$\nu_4 = 3 \times 10^{-8}$

initial conditions

$\mathcal{E}(0) = 10^{-3}$

$j = 1, 5$, $\beta = 0.5$

integration

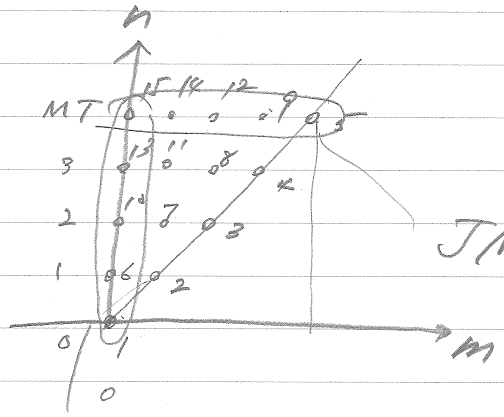
$\Delta t = 0.2$

$0 \le t \le 200$ ← IMAXP = 10, IMAXE = 100, ISTPE = 10, ISDEL = 10

12,000

$$\mathcal{E}(t) = \frac{1}{2} \sum_n \sum_m n(n+1) \left| \psi_n^m(t) \right|^2$$

三角切断における配列の1次元化



← MT = 4 の場合

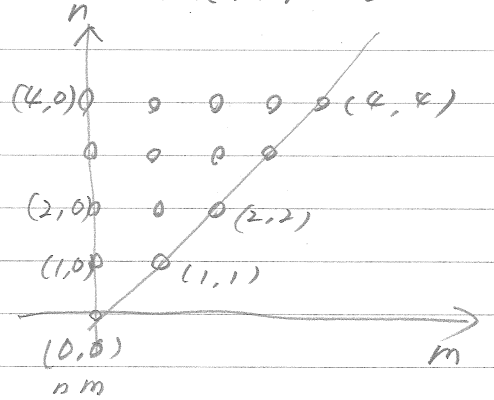
JMIN

JMIN(0:MT)
JMAX(0:MT)

JMIN(0) = 1
JMAX(0) = MT + 1

J = 1 ~ MT
  JMIN(J) = JMAX(J-1) + 1
  JMAX(J) = JMIN(J) + MT - J

ZP(N, M)




```
DO L = 0, MT
   M = -1
   DO J = JMIN(L), JMAX(L)
      M = M + 1
      N = M + L

      ZOUT(J) = ZP(N, M)
CONT
```

▨ 無次元化速度

$$u^* = -\frac{\sqrt{1-\mu^2}}{a^*}\frac{\partial\psi^*}{\partial\mu}$$

$$= -\frac{1}{a^*}\frac{L^2}{T}\sqrt{1-\mu^2}\frac{\partial\psi}{\partial\mu}$$

$$= -\gamma U\sqrt{1-\mu^2}\frac{\partial\psi}{\partial\mu}$$

$$v^* = \frac{1}{a^*\sqrt{1-\mu^2}}\frac{\partial\psi^*}{\partial\lambda}$$

$$= \gamma U\frac{1}{\sqrt{1-\mu^2}}\frac{\partial\psi}{\partial\lambda}$$

$$\therefore\quad (u^*, v^*) = \gamma U(u, v)$$

$$u = -\sqrt{1-\mu^2}\frac{\partial\psi}{\partial\mu}$$

$$v = \frac{1}{\sqrt{1-\mu^2}}\frac{\partial\psi}{\partial\lambda}$$

・角運動量：

$$a^*\cos\phi\, u^* = a^*\sqrt{1-\mu^2}\times\left(-\gamma U\sqrt{1-\mu^2}\right)\frac{\partial\psi}{\partial\mu}$$

$$= -\gamma a^* U(1-\mu^2)\frac{\partial\psi}{\partial\mu}$$

◎ 渦度

$$\omega^* = \Delta^* \psi^* = \frac{1}{a^{*2}} \frac{L^2}{T} \Delta\psi = r^{2} \cdot \frac{1}{T} \Delta\psi$$

$$\omega = \Delta\psi$$

・絶対渦度：

$$\omega + 2\Omega\mu$$

◎ 運動エネルギー

$$\frac{1}{2}|u^*|^2 = \frac{1}{2}(u^{*2} + v^{*2})$$

$$E(\lambda, \mu, t) = \frac{1}{2}(u^2 + v^2)$$

◎ エンストロフィー

$$\omega^{*2} = r^4 \frac{1}{T^2} (\Delta\psi)^2$$

$$\omega^2 = (\Delta\psi)^2$$

ⓘ 記号:

zonal mean: $\overline{a} = \frac{1}{2\pi}\int_0^{2\pi} a\, d\lambda$

deviation from zonal mean: $a' = a - \overline{a}$

ensemble (time) mean: $\langle a \rangle = \frac{1}{N}\sum_{n=1}^{N} a_n$

deviation from ensemble mean: $a^* = a - \langle a \rangle$

ⓘ momentum (← grad ψ)

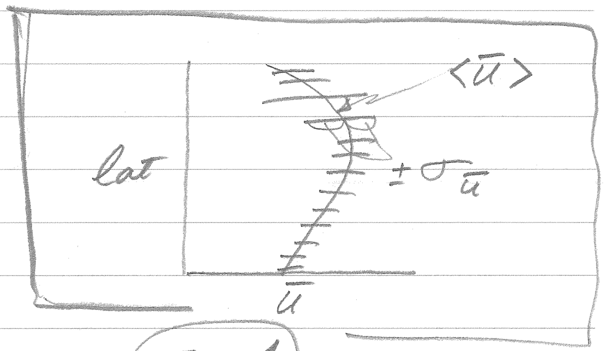
① zonal mean

CZMU

1-1 $\overline{u}(\phi, t)$

($\overline{v} = 0$)

$\langle \overline{u} \rangle (\phi)$ → Fig. 1



$\overline{u}^* (\phi, t)$ ⟶ $\sigma_{\overline{u}}(\phi) = \sqrt{\langle \overline{u}^{*2} \rangle}$

② deviation from z.m.

2-1 $u'(\lambda, \phi, t)$

r-m-s u' → $\delta u(\phi, t) \equiv \sqrt{\overline{u'^2}}$

$\langle \delta u \rangle (\phi)$, $\sigma_{\delta u}(\phi) = \sqrt{\langle \delta u^{*2} \rangle}$ → Fig. 2

NO. ②

2-2 $v'(\lambda, \phi, t)$

$\langle \delta v \rangle(\phi)$, $\sigma_{\delta v}(\phi)$ — Fig. 3

2-3 ratio $\langle \delta v \rangle / \langle \delta u \rangle$ "渦"の等方性.

$$r(\phi) = \frac{\langle \delta v \rangle(\phi)}{\langle \delta u \rangle(\phi)}$$

Fig. 4

(III) energy ($\propto |\mathrm{grad}\,\psi|^2$)

$$E(\lambda, \phi, t) = \frac{1}{2}(u^2 + v^2)$$

① zonal mean

3-1 $\overline{E}(\phi, t)$

$\langle \overline{E} \rangle(\phi)$, $\sigma_{\overline{E}}(\phi)$ — Fig. 5

② deviation from z.m.

4-1 $E'(\lambda, \phi, t)$

$$\delta E(\phi, t) = \sqrt{\overline{E'^2}}$$

$\langle \delta E \rangle(\phi)$, $\sigma_{\delta E}(\phi)$ — Fig. 6

2-4

< r.m.s. v / r.m.s. u >

$$\text{r.m.s. } v = \sqrt{\overline{v^2}} = \delta v$$

$$\text{r.m.s. } u = \sqrt{\overline{u^2}} = \sqrt{\bar{u}^2 + \overline{u'^2}}$$

$$= \sqrt{\bar{u}^2 + (\delta u)^2}$$

NO. ③

Ⓘ vorticity ($\leftarrow \Delta\psi$)

$$\omega(\lambda, \phi, t) = \Delta\psi$$

① zonal mean

[5-1] $\overline{\omega}(\phi, t)$

$\langle \overline{\omega} \rangle(\phi)$, $\sigma_{\overline{\omega}}(\phi)$ (Fig. 7)

② deviation from z.m

[6-1] $\omega'(\lambda, \phi, t)$ $\rightarrow \delta\omega(\phi, t) = \sqrt{\overline{\omega'^2}}$

$\langle \delta\omega \rangle(\phi)$, $\sigma_{\delta\omega}(\phi)$ (Fig. 8)

Ⓘ enstrophy ($\leftarrow (\Delta\psi)^2$)

$$Q(\lambda, \phi, t) = (\Delta\psi)^2$$

① zonal mean

[7-1] $\overline{Q}(\phi, t)$

$\langle \overline{Q} \rangle(\phi)$, $\sigma_{\overline{Q}}(\phi)$ (Fig. 9)

② deviation from z.m

[8-1] $Q'(\lambda, \phi, t)$ $\rightarrow \delta Q(\phi, t) = \sqrt{\overline{Q'^2}}$

$\langle \delta Q \rangle(\phi)$, $\sigma_{\delta Q}(\phi)$ (Fig. 10)

角運動量：

$$M(\phi, t) = a \cos\phi \, \bar{u}(\phi, t)$$

渦度：

$$\omega = e_r \cdot \mathrm{rot}\, u$$

$$= \frac{1}{a\cos\phi} \left\{ \frac{\partial v}{\partial \lambda} - \frac{\partial}{\partial \phi}(\cos\phi \, u) \right\}$$

$$\bar{\omega} = \frac{-1}{a\cos\phi} \frac{\partial}{\partial \phi}(\cos\phi \, \bar{u}) \qquad \bar{u} = -\frac{\sqrt{1-\mu^2}}{a} \frac{d\bar{\psi}}{d\mu} = -\frac{1}{a} \frac{d\bar{\psi}}{d\phi}$$

$$M = \mathrm{const} \longleftrightarrow \bar{\omega} = 0 \quad //$$

Rayleigh 条件

帯状流が不安定となる必要条件：

「$-1 < \mu < 1$ で $d\bar{\omega}/d\mu + 2\Omega$ が符号をかえる」

$$\frac{d\bar{\omega}}{d\mu} = \frac{1}{\cos\phi}\frac{d}{d\phi}\left\{ \frac{-1}{a\cos\phi}\frac{d}{d\phi}\left(-\frac{\cos\phi}{a}\frac{d\bar{\psi}}{d\phi}\right)\right\}$$

$\mu = \sin\phi$

$$= \frac{1}{\cos\phi}\frac{d}{d\phi}\left\{ \frac{1}{a^2\cos\phi}\frac{d}{d\phi}\left(\cos\phi\frac{d\bar{\psi}}{d\phi}\right)\right\}$$

$d\mu = \cos\phi \, d\phi$

$$= \frac{1}{a^2\cos\phi}\frac{d}{d\phi}\left\{ \frac{1}{\cos\phi}\left(-\sin\phi\frac{d\bar{\psi}}{d\phi} + \cos\phi\frac{d^2\bar{\psi}}{d\phi^2}\right)\right\}$$

$$= \frac{1}{a^2\cos\phi}\frac{d}{d\phi}\left(\frac{d^2\bar{\psi}}{d\phi^2} - \tan\phi\frac{d\bar{\psi}}{d\phi}\right)$$

$\langle \bar{\omega} + 2\Omega\mu \rangle$ → 単増 : stable

coop Filing Leaf 7mm×35行

R/mines

$$k_\beta = \sqrt{\beta / 2U}$$

$$\beta = \frac{1}{a}\frac{df}{d\phi} = \frac{2\Omega}{a}\cos\phi$$

U : r.m.s. particle speed

- $\frac{1}{2}U^2 = \mathcal{E}(0) = 1$

  $U = \sqrt{2}$

- $a = 1$

$$k_\beta = \sqrt{\Omega \cos\phi / \sqrt{2}}$$

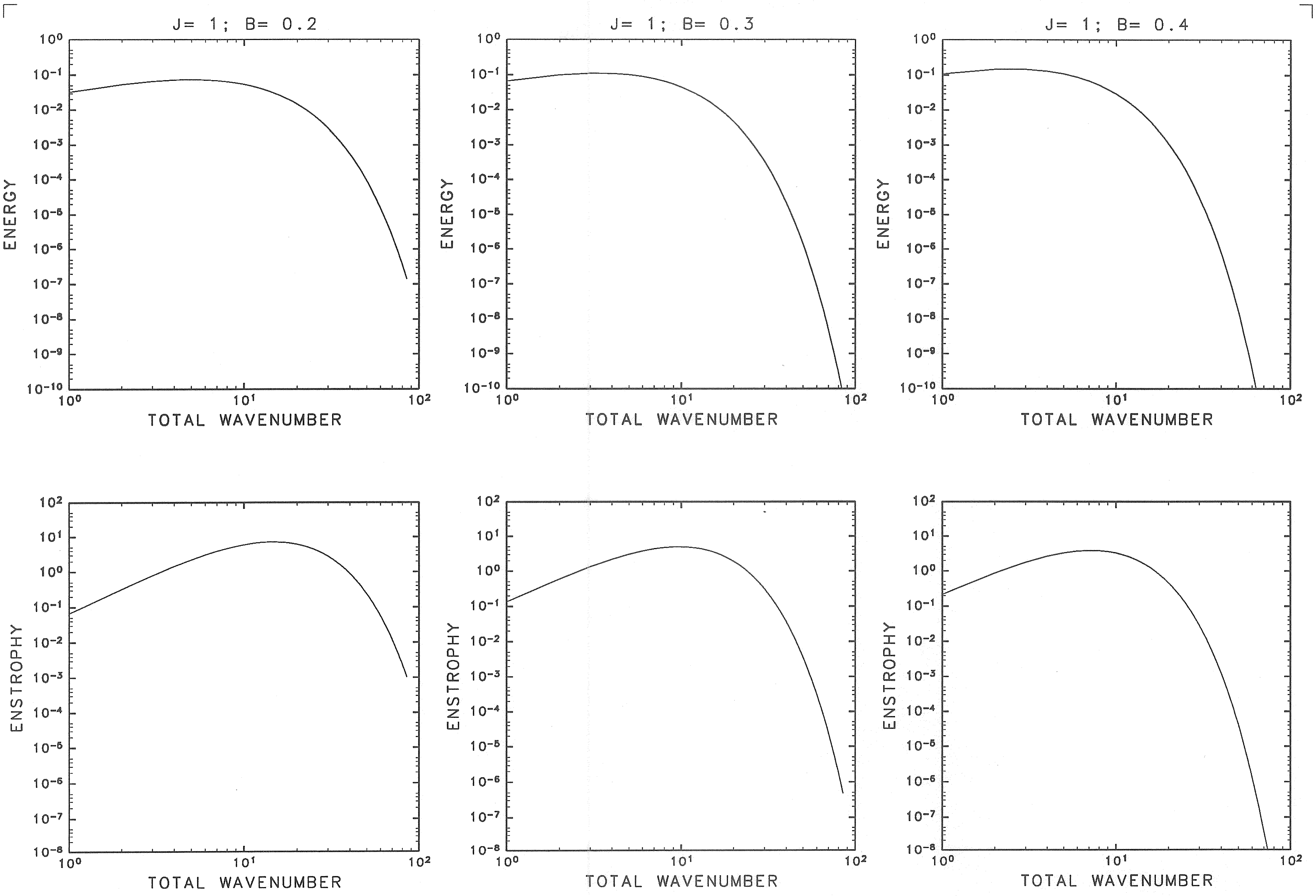
赤道 : $\cos\phi = 1$

$$k_\beta = \sqrt{\Omega} / \sqrt[4]{2}$$

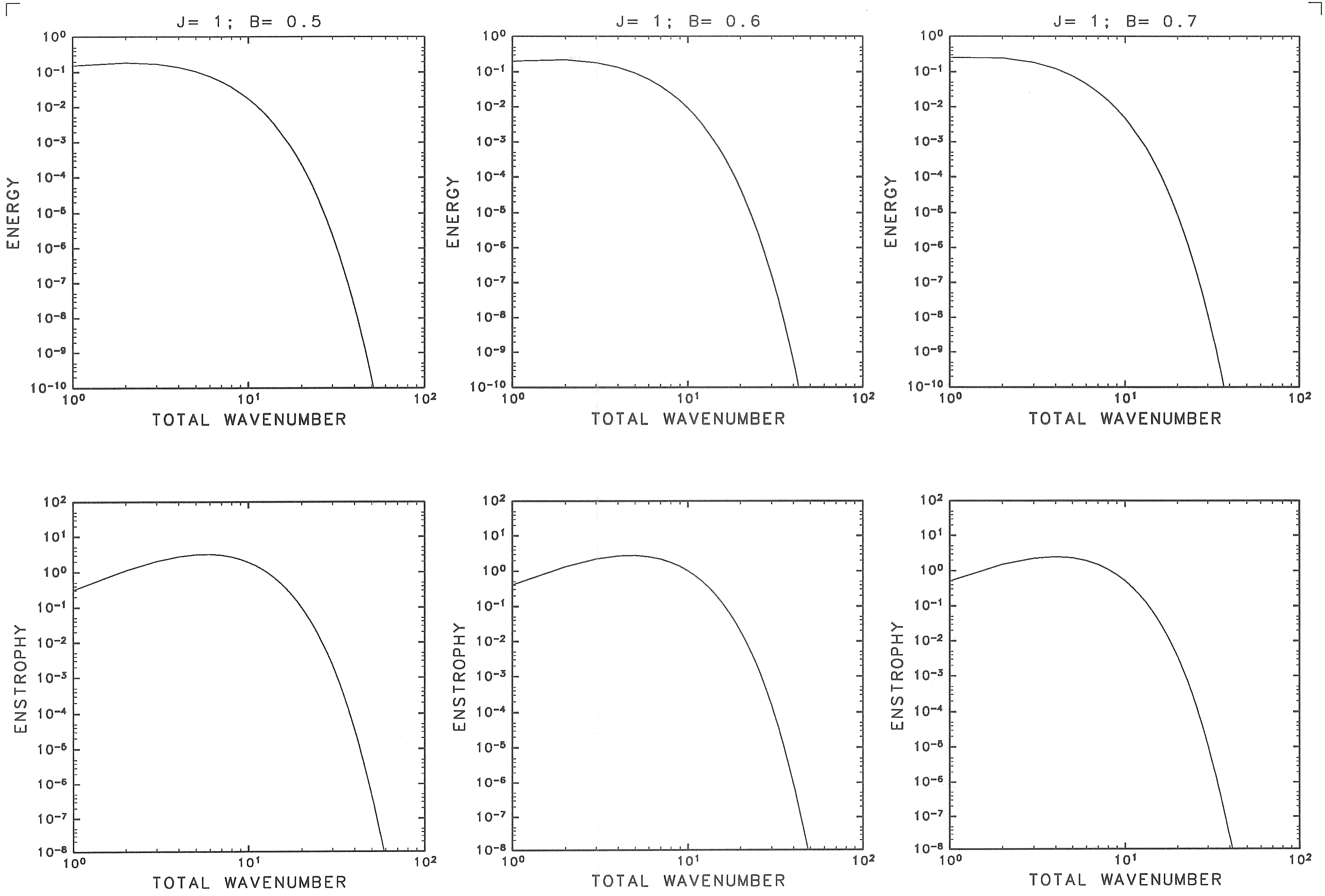
Ω	0	25	50	100	200	400
k_β	0	4.2	5.9	8.4	11.8	16.8

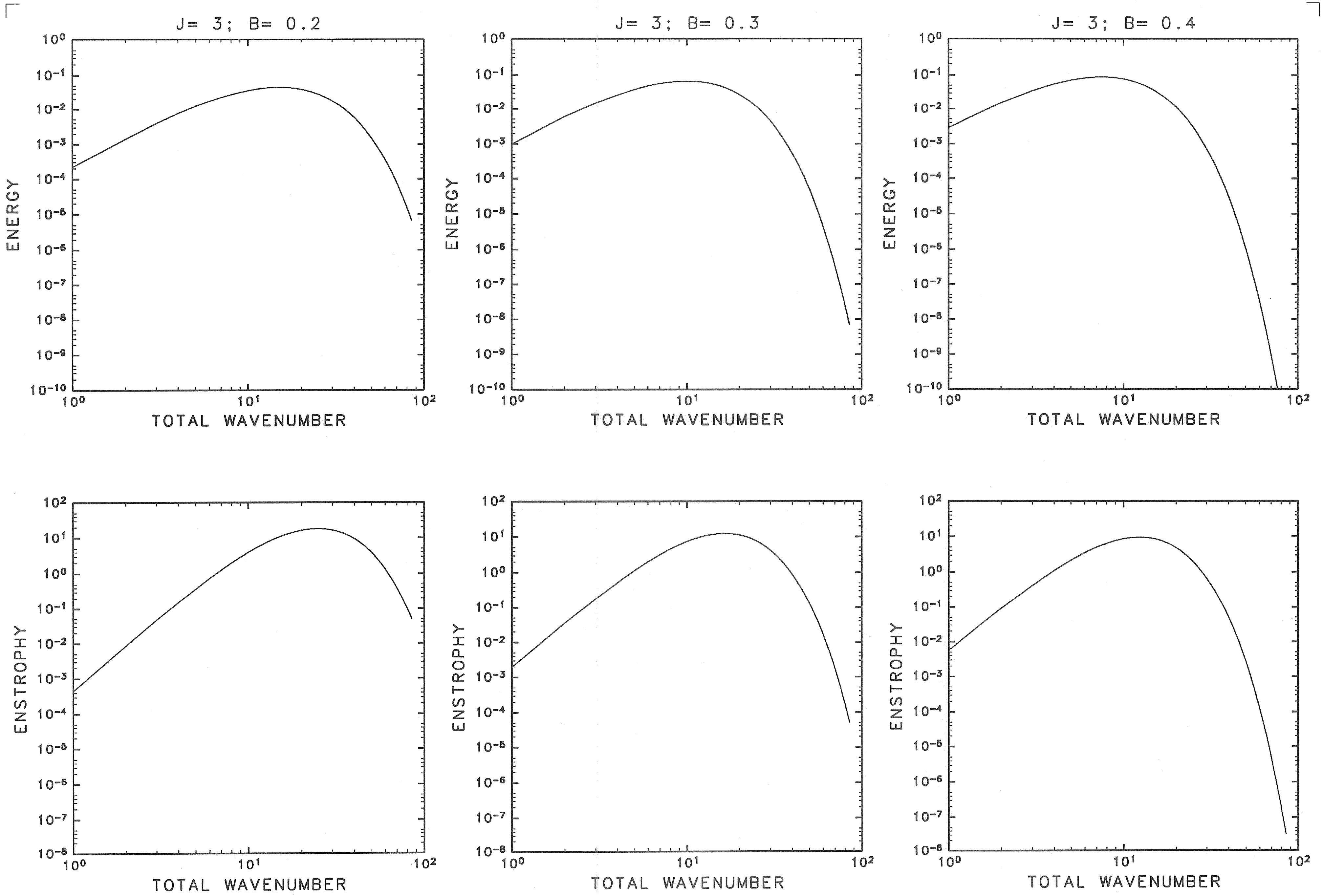
30° : $\cos\phi = 0.866$

k_β 0

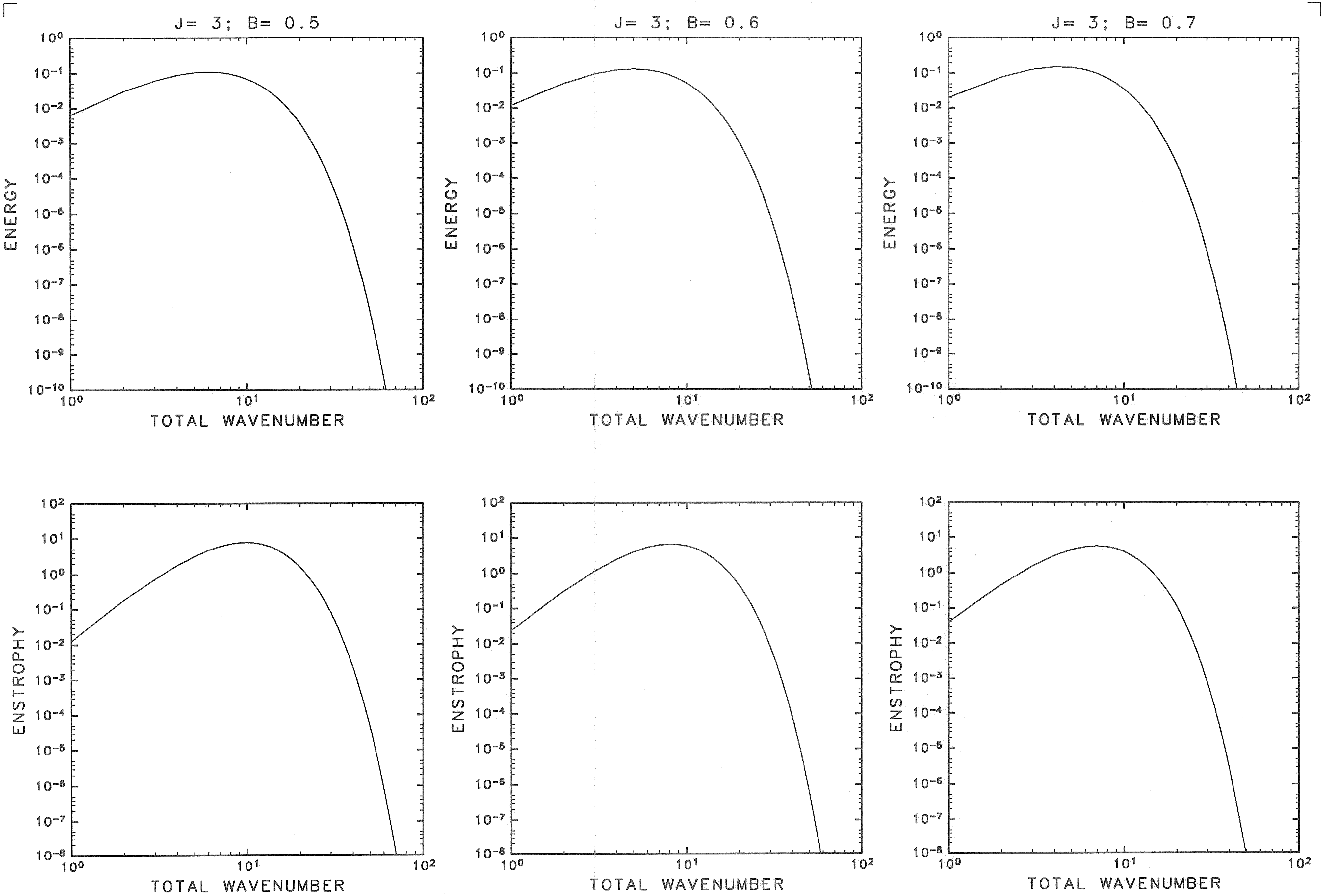


J= 1; B= 0.2
J= 1; B= 0.3
J= 1; B= 0.4
ENERGY
ENSTROPHY
TOTAL WAVENUMBER
10⁰
10¹
10²

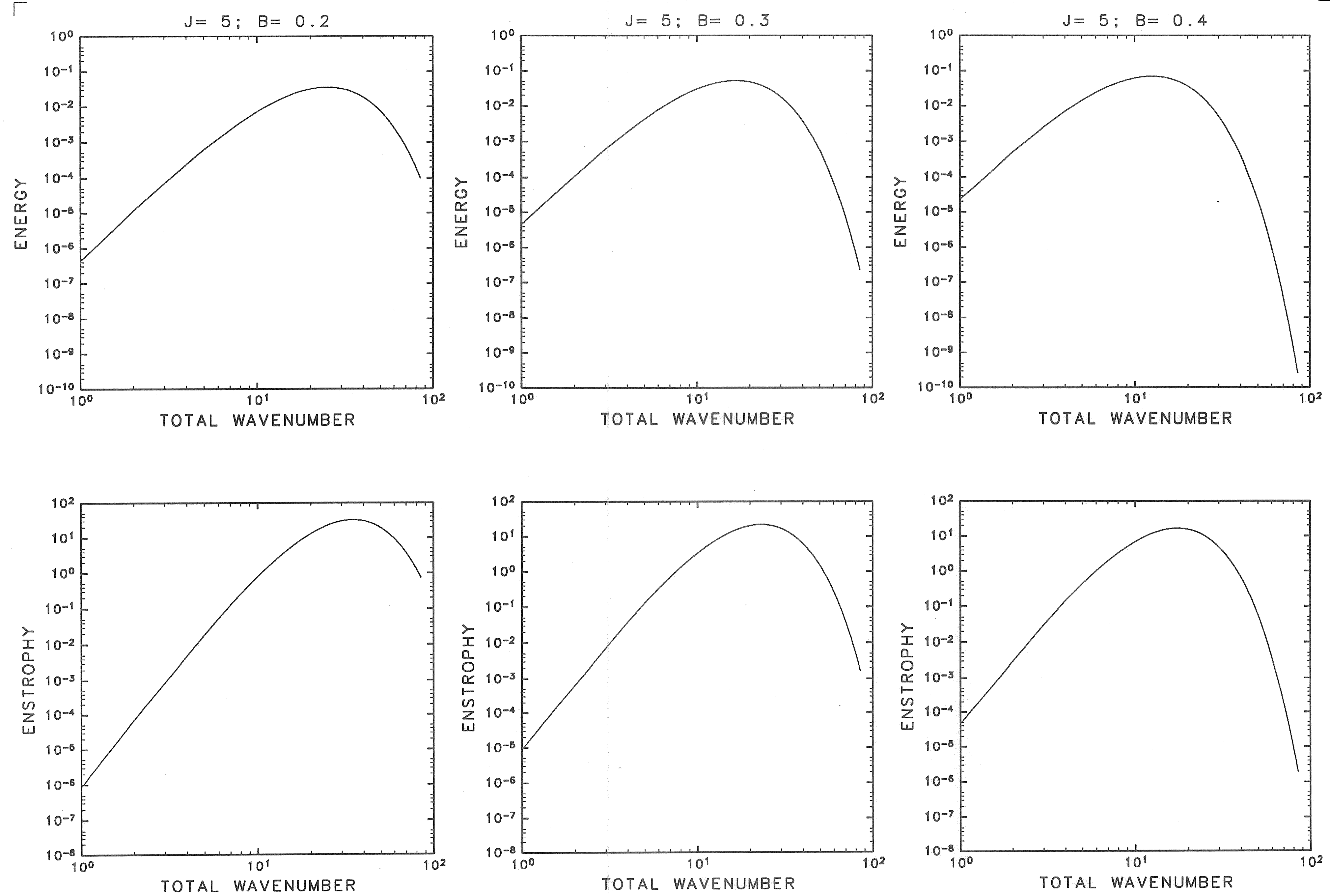

J= 1; B= 0.5
J= 1; B= 0.6
J= 1; B= 0.7
ENERGY
ENSTROPHY
TOTAL WAVENUMBER
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{2}
10^{1}

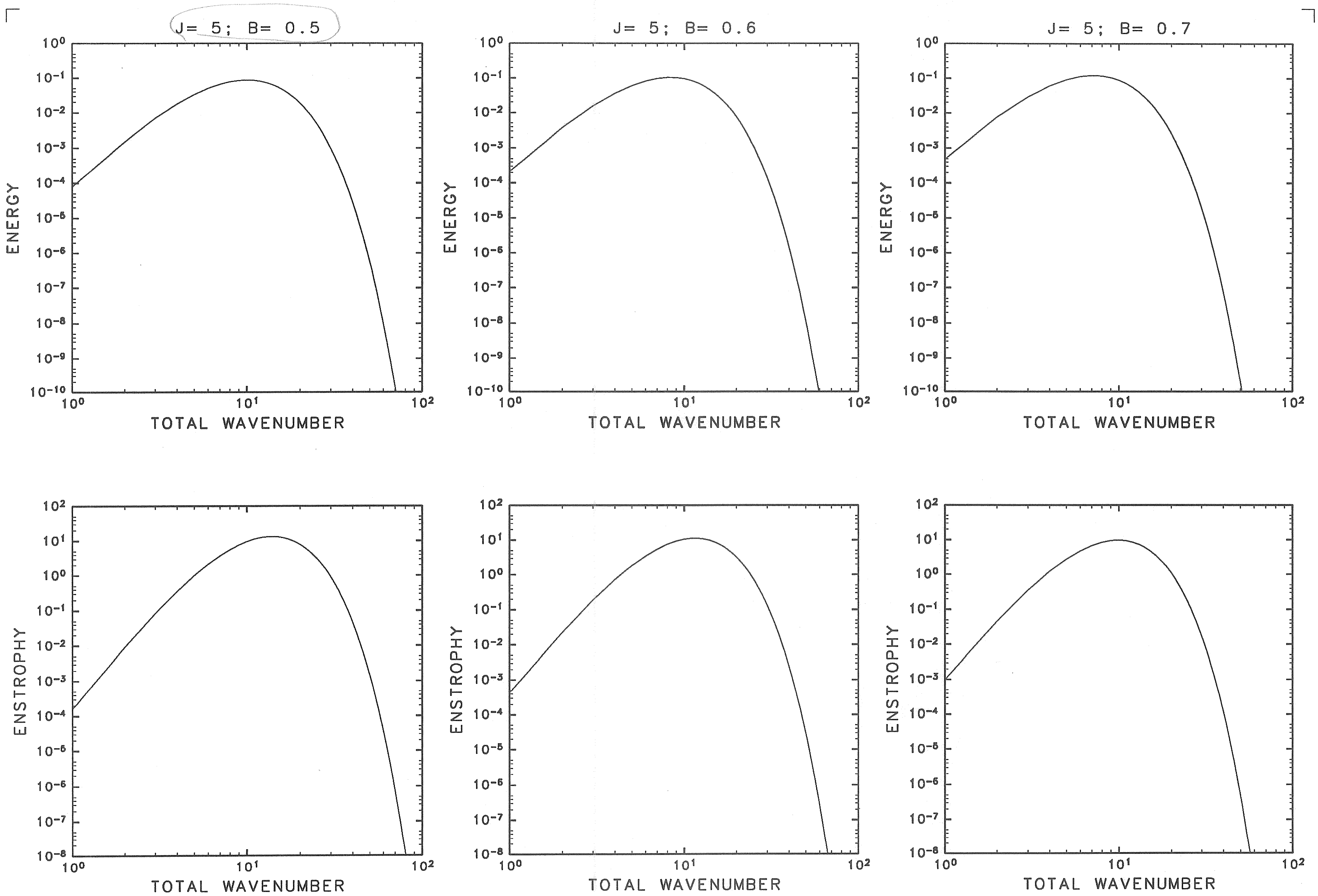


J= 3; B= 0.2
J= 3; B= 0.3
J= 3; B= 0.4
ENERGY
ENSTROPHY
TOTAL WAVENUMBER
10^{0} 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10}
10^{2} 10^{1} 10^{0} 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8}
10^{0} 10^{1} 10^{2}

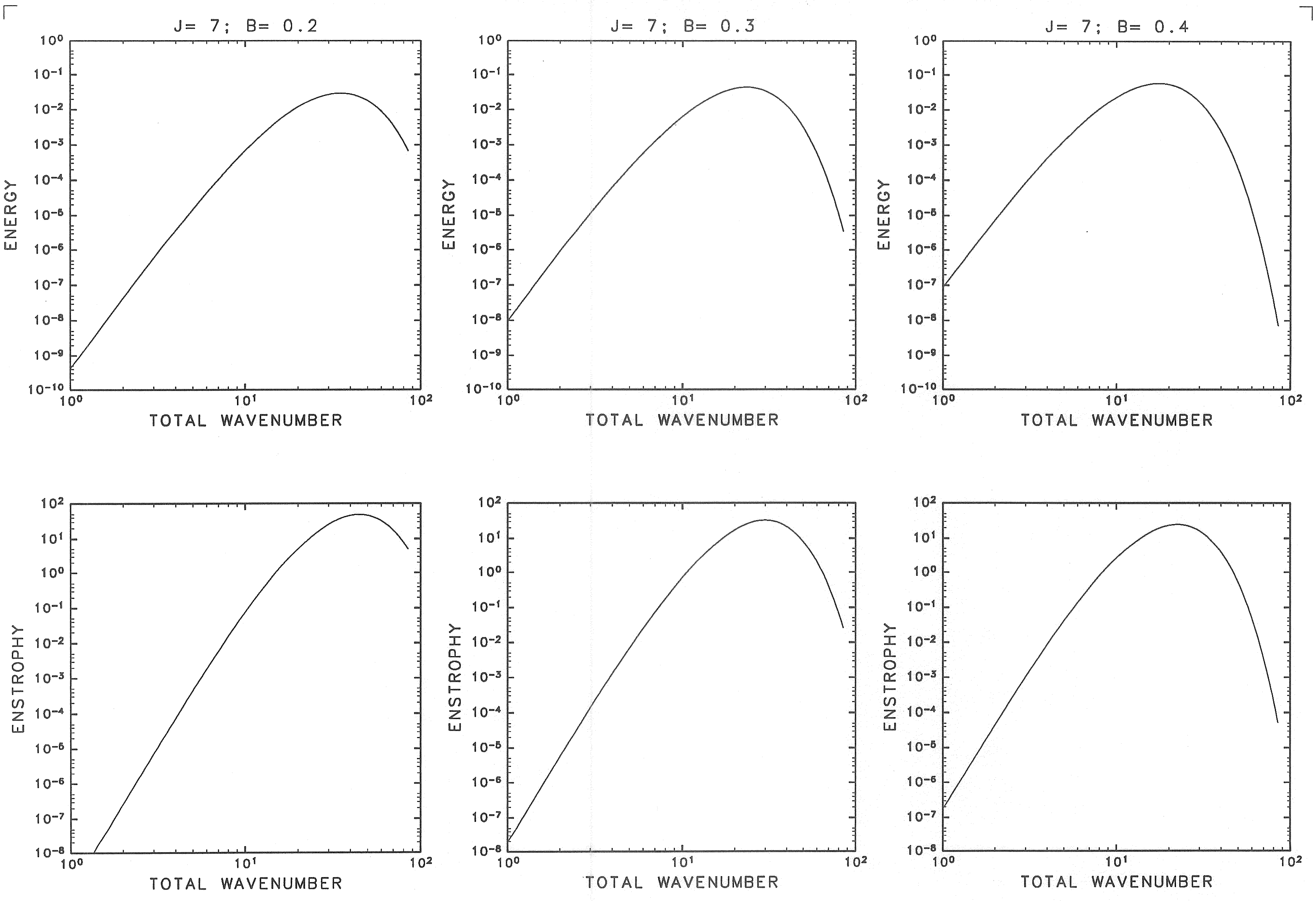


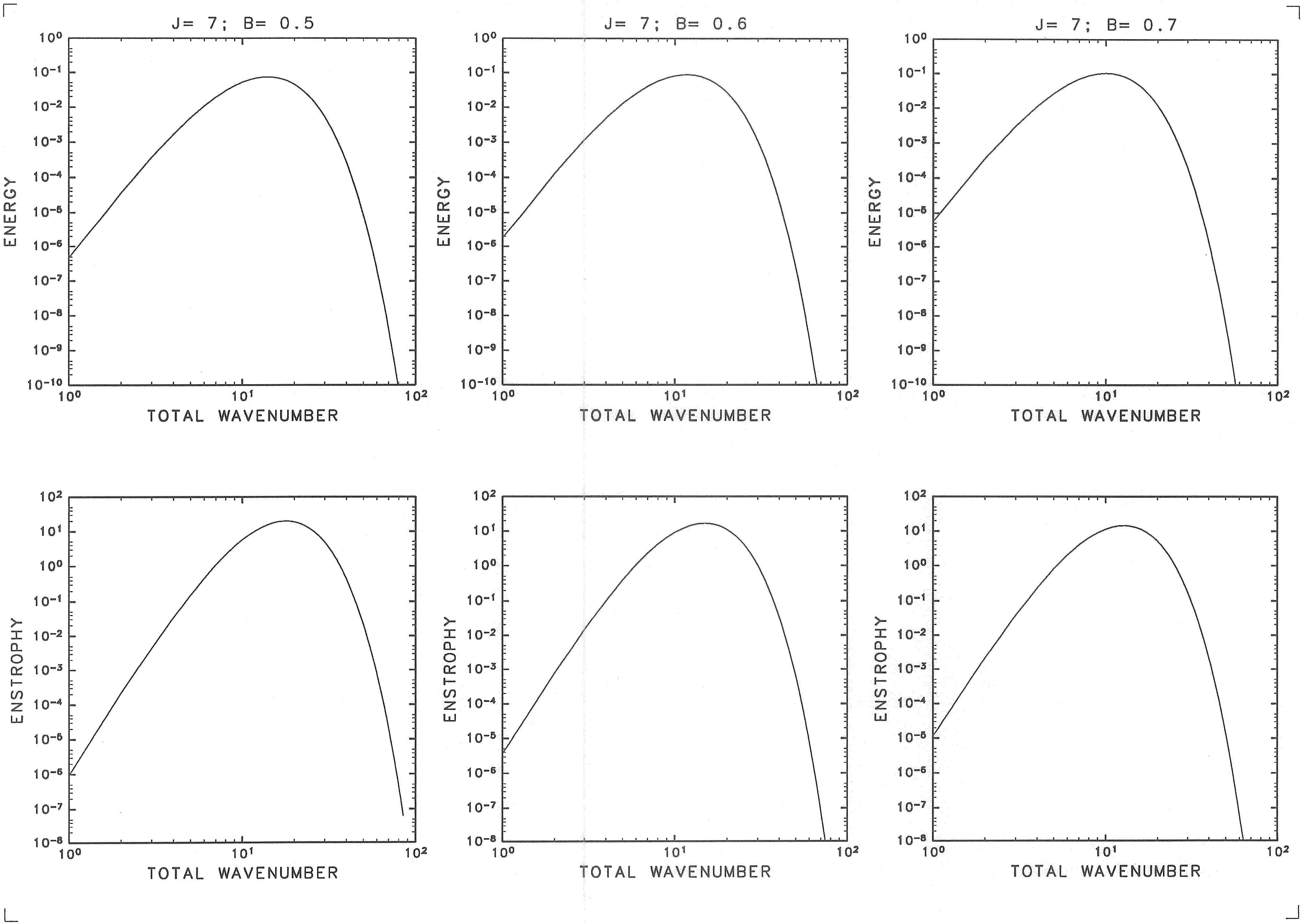
J= 3; B= 0.5
J= 3; B= 0.6
J= 3; B= 0.7
ENERGY
ENSTROPHY
TOTAL WAVENUMBER


J= 5; B= 0.2
J= 5; B= 0.3
J= 5; B= 0.4
ENERGY
ENSTROPHY
TOTAL WAVENUMBER



J= 5; B= 0.5
J= 5; B= 0.6
J= 5; B= 0.7
ENERGY
ENSTROPHY
TOTAL WAVENUMBER


J= 7; B= 0.2
J= 7; B= 0.3
J= 7; B= 0.4
ENERGY
ENSTROPHY
TOTAL WAVENUMBER
10^{0}
10^{1}
10^{2}
10^{-10}


J= 7; B= 0.5
J= 7; B= 0.6
J= 7; B= 0.7
ENERGY
ENSTROPHY
TOTAL WAVENUMBER

ginit.f

```
<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.40134E-01      J = 1      B = 0.20
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.16053E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.78927E-01
   TIME SCALE     = 0.78927E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.90677E-01      J = 1      B = 0.30
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.73885E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.11634E+00
   TIME SCALE     = 0.11634E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.16214E+00      J = 1      B = 0.40
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.43070E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.15237E+00
   TIME SCALE     = 0.15237E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.25525E+00      J = 1      B = 0.50
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.28589E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.18702E+00
   TIME SCALE     = 0.18702E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.37093E+00      J = 1      B = 0.60
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.20608E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.22028E+00
   TIME SCALE     = 0.22028E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.51034E+00      J = 1      B = 0.70
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.15730E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.25214E+00
   TIME SCALE     = 0.25214E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.26668E-03      J = 3      B = 0.20
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.51970E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.43866E-01
   TIME SCALE     = 0.43866E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.13500E-02      J = 3      B = 0.30
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.23555E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.65156E-01
   TIME SCALE     = 0.65156E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>
```

```
   A = 0.42665E-02      J = 3      B = 0.40
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.13500E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.86068E-01
   TIME SCALE     = 0.86068E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.10416E-01      J = 3      B = 0.50
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.87993E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.10660E+00
   TIME SCALE     = 0.10660E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.21596E-01      J = 3      B = 0.60
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.62212E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.12678E+00
   TIME SCALE     = 0.12678E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.40005E-01      J = 3      B = 0.70
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.46517E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.14662E+00
   TIME SCALE     = 0.14662E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.53367E-06      J = 5      B = 0.20
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.10753E+04
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.30496E-01
   TIME SCALE     = 0.30496E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.60750E-05      J = 5      B = 0.30
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.48666E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.45330E-01
   TIME SCALE     = 0.45330E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.34133E-04      J = 5      B = 0.40
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.27750E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.60030E-01
   TIME SCALE     = 0.60030E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.13021E-03      J = 5      B = 0.50
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.18000E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.74536E-01
   TIME SCALE     = 0.74536E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.38880E-03      J = 5      B = 0.60
   TOTAL ENERGY = 0.10000E+01      TOTAL ENSTROPHY = 0.12667E+03
```

```
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.88852E-01
   TIME SCALE     = 0.88852E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.98041E-03      J = 5      B = 0.70
   TOTAL ENERGY = 0.10000E+01       TOTAL ENSTROPHY = 0.94286E+02
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.10299E+00
   TIME SCALE     = 0.10299E+00

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.51054E-09      J = 7      B = 0.20
   TOTAL ENERGY = 0.10000E+01       TOTAL ENSTROPHY = 0.18041E+04
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.23543E-01
   TIME SCALE     = 0.23543E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.13018E-07      J = 7      B = 0.30
   TOTAL ENERGY = 0.10000E+01       TOTAL ENSTROPHY = 0.82656E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.34783E-01
   TIME SCALE     = 0.34783E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.13003E-06      J = 7      B = 0.40
   TOTAL ENERGY = 0.10000E+01       TOTAL ENSTROPHY = 0.47000E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.46127E-01
   TIME SCALE     = 0.46127E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.77505E-06      J = 7      B = 0.50
   TOTAL ENERGY = 0.10000E+01       TOTAL ENSTROPHY = 0.30400E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.57354E-01
   TIME SCALE     = 0.57354E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.33326E-05      J = 7      B = 0.60
   TOTAL ENERGY = 0.10000E+01       TOTAL ENSTROPHY = 0.21333E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.68465E-01
   TIME SCALE     = 0.68465E-01

<<< DYNAMICAL CHARACTERISTICS OF INITIAL STATE >>>

   A = 0.11438E-04      J = 7      B = 0.70
   TOTAL ENERGY = 0.10000E+01       TOTAL ENSTROPHY = 0.15837E+03
   VELOCITY SCALE = 0.10000E+01
   LENGTH SCALE   = 0.79463E-01
   TIME SCALE     = 0.79463E-01
```

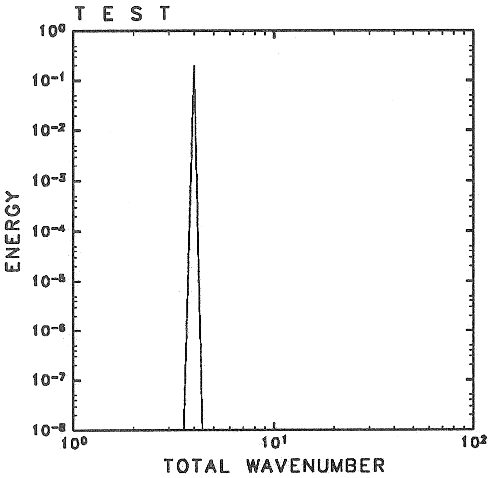
```
      PROGRAM EINIT
***********************************************************************
*
*     SET GAUSSIAN LATITUDES AND LEGENDRE FUNCTIONS
*
*        This program computes Gaussian latitudes and Legendre functions
*        at the latitides.
*
*        coded by S. Yoden(Kyoto Univ.)
*        May. 1990
***********************************************************************
      IMPLICIT REAL* 8(A-H,O-Z)

* transform resolution parameters
* grid resolution
      PARAMETER ( NLON= 128, NLAT= 64, NLOH=NLON/2, NLAH=(NLAT+1)/2 )
* spectral resolution
      PARAMETER ( MT= 42 )
* parameters
* NLON = number of longitudes
* NLAT = number of latitudes
* NLAH = number of latitudes between pole and equator
* MT = maximum zonal wavenumber for triangle truncation

      DIMENSION COLAT(NLAT), GW(NLAT)
      DIMENSION DCCLT(NLAT)
      DIMENSION PMN(NLAT,0:MT,0:MT), DPMN(NLAT,0:MT,0:MT)

*             COLAT : colatitude(deg) of the Gaussian grids
*             GW    : Gaussian weight
*             DCCLT : double precision of cos(COLAT)
*             PMN   : normalized Legendre functions
*             DPMN  : d(PMN)/d(DCCLT)

      CHARACTER WDATA*50
      DATA WDATA/'xxxx'/
*-----------------------------------------------------------------------
* computation of Gaussian latitudes
      CALL GAUSS( COLAT, GW, DCCLT )

* computation of Legendre functions
      CALL LGNDR( DCCLT, GW, PMN, DPMN )

* storage of Gaussian weights and Legendre functions
C      OPEN (1, FILE=WDATA, FORM='UNFORMATTED')
C      WRITE(1) MT, NLAT, COLAT, GW, PMN, DPMN
C      CLOSE(1)

      STOP
      END
***********************************************************************
      SUBROUTINE GAUSS( DCOLAT, DGW, DCCLT )
*
*        GAUSS gives the Gaussian latitudes by finding the roots
*        of the ordinary Legendre polynomial of degree NLAT
*        using Newton's iteration method.
*        It also computes the associated Gaussian weights.
*        The solutions are symmetric with respect to the equator
*        so that only half of the whole domain is computed.
*
*        On exit: for each Gaussian latitude
*           DCOLAT - the colatitudes in radians
*           DGW    - the Gaussian weights
*           DCCLT  - quadruple precision of cos(colatitude)
***********************************************************************
```

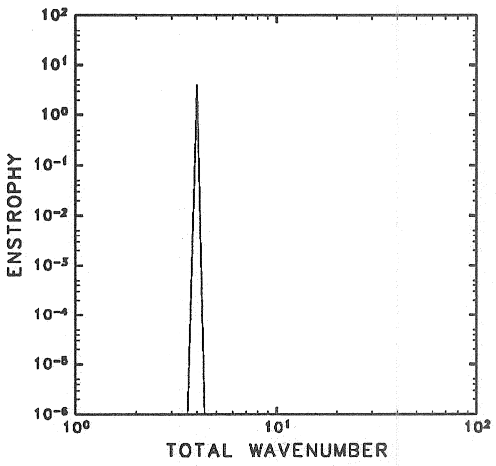
```
      IMPLICIT REAL* 8(A-H,O-Z)

* transform resolution parameters
* grid resolution
      PARAMETER ( NLON= 128, NLAT= 64, NLOH=NLON/2, NLAH=(NLAT+1)/2 )
* spectral resolution
      PARAMETER ( MT= 42 )

      DIMENSION DCOLAT(NLAT), DGW(NLAT)
      DIMENSION DCCLT (NLAT)
*-----------------------------------------------------------------------
      PI     = 4.D0*ATAN(1.D0)
      RADINV = 180.D0/PI

* convergence criterion for iteration
      EPS    = 1.D-15
      XPREV  = 1.D10

* loop over latitudes, from pole to equator ----------------------------
      DO 10 L=1,NLAH

* set first guess
        X = SIN( PI * (NLAT+1.D0-2.D0*L) / (2.D0*NLAT+1.D0) )

* Newton method
        ITRMAX = 500
        DO 20 ITR=1,ITRMAX
          PJ0 = 1.D0
          PJ1 = X

          DO 30 J=1,NLAT-1
            PJ2 = ( (2.D0*J+1.D0)*X*PJ1 - J*PJ0 ) / (J+1.D0)
            PJ0 = PJ1
            PJ1 = PJ2
30        CONTINUE

          DPJ = NLAT*(PJ0-X*PJ1)/(1.D0-X*X)

          X = X - PJ1/DPJ

* check the criterion for convergence
          IF(ABS(X-XPREV) .LE. EPS) GO TO 1
          IF(ITR .EQ. ITRMAX) GO TO 999

          XPREV = X
20      CONTINUE
1       CONTINUE
* the criterion for convergence is fulfilled
* determine DCCLT, DCOLAT
        DCCLT (L) = X
        DCOLAT(L) = ACOS(X)*RADINV

* computation of the Gaussian weight
        DGW(L) = 2.D0 * (1.D0-X*X) / (NLAT*NLAT*PJ0*PJ0)
10    CONTINUE
*-----------------------------------------------------------------------

* determine the southern hemisphere values by symmetry
        DO 40 L=1,NLAH
          DCOLAT(NLAT-L+1) = - DCOLAT(L) + 180.D0
          DCCLT (NLAT-L+1) = - DCCLT (L)
          DGW   (NLAT-L+1) =   DGW   (L)
40      CONTINUE

* print out ------------------------------------------------------------
      WRITE(6,600)
```

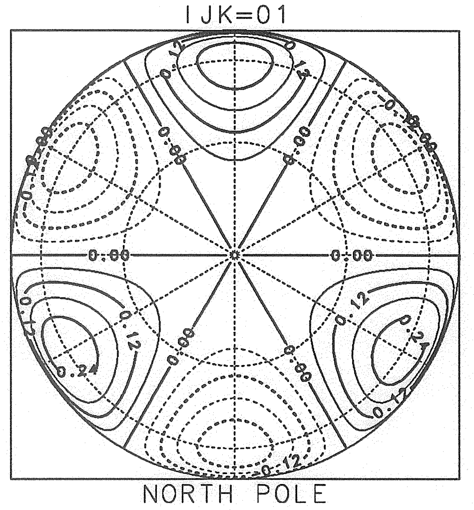
```fortran
600   FORMAT(25X, 'COS(COLATITUDE)', 25X, 'COLATITUDE(DEG)',
     +        25X, 'GAUSSIAN WEIGHT' )
      WRITE(6,610)(DCCLT(L),DCOLAT(L),DGW(L), L=1,NLAT)
610   FORMAT( 5X, F35.30, 5X, F35.30, 5X, F35.30)

      SUM = 0.D0
      DO 50 L=1,NLAT
        SUM = SUM + DGW(L)
50    CONTINUE

      WRITE(6,620) SUM
620   FORMAT(//10X,'SUM OF WEIGHTS = ', F40.35)

      RETURN

*-----------------------------------------------------------------------
* not converged
999   CONTINUE
      WRITE(6,699) L
699   FORMAT(10X, 'COMPUTATION DID NOT CONVERGE AT SUBROUTINE GAUSS'/
     +        10X, 'L = ', I3)
      WRITE(6,698) X, XPREV
698   FORMAT(10X, 'X = ', F36.30, 5X, 'XPREV = ', F36.30)
      STOP 999

      END
***********************************************************************
      SUBROUTINE LGNDR( DCCLT, GW, PMN, DPMN )
*
*        LGNDR generates the values of associated Legendre functions
*        and their derivatives  for given Gaussian latitude.
*
*        On entry:
*           DCCLT - cos(Gaussian colatitude)
*           GW    - Gaussian weights
*
*        On exit:
*           PMN  - normalized Legendre functions
*           DPMN - d(PMN)/d(COSCLT)
*  for each Gaussian latitude
***********************************************************************

      IMPLICIT REAL* 8(A-H,O-Z)

* transform resolution parameters
* grid resolution
      PARAMETER ( NLON= 128, NLAT= 64, NLOH=NLON/2, NLAH=(NLAT+1)/2 )
* spectral resolution
      PARAMETER ( MT= 42 )

      DIMENSION DCCLT(NLAT), GW(NLAT)
      DIMENSION DP (0:MT,0:MT+1),     DDP (0:MT,0:MT)
      DIMENSION PMN(NLAT,0:MT,0:MT), DPMN(NLAT,0:MT,0:MT)
*-----------------------------------------------------------------------
      DO 10 J=1,NLAT
        CALL LGNDR1( DCCLT(J), DP, DDP )

        DO 20 M=0,MT
        DO 20 N=M,MT
          PMN (J,N,M) = DP (M,N)
          DPMN(J,N,M) = DDP(M,N)     / (DCCLT(J)*DCCLT(J)-1.D0)
20      CONTINUE
10    CONTINUE

* print out(including some checking routines)
      WRITE(6,'(1H1)')
```

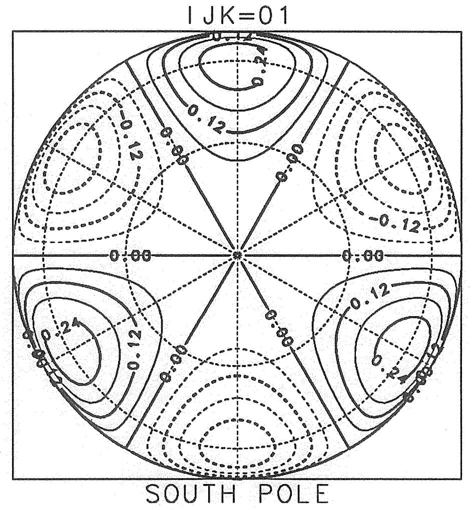
```
      DO 30 M=0,MT
      DO 30 N=M,MT
        SUMP  = 0.D0
        SUMDP = 0.D0

        DO 40 J=1,NLAT
          SUMP  = SUMP  +  PMN(J,N,M)* PMN(J,N,M)*GW(J)
          SUMDP = SUMDP + DPMN(J,N,M)*DPMN(J,N,M)*GW(J)
40      CONTINUE

        WRITE(6,'(/)')
        DO 50 J=1,NLAT
          WRITE(6,610) J, M, N, PMN(J,N,M), DPMN(J,N,M)
610       FORMAT('  J=', I3, ' (M,N)=(', I3, ',', I3, ')  PMN = ',
     +            F13.8, '  DPMN = ', F13.8 )
50      CONTINUE

        WRITE(6,620) SUMP, SUMDP
620     FORMAT(1X, 'SUMP = ', F36.30, '    SUMDP = ', F40.30 )
30    CONTINUE

      RETURN
      END
*************************************************************************
      SUBROUTINE LGNDR1( X, P, DP )
*
*        LGNDR1 generates the values of associated Legendre functions
*        and their derivatives  for a given latitude.
*
*        On entry:
*           X  - cos(Gaussian colatitude)
*
*        On exit:
*           P  - normalized Legendre functions
*           DP - (X**2-1) * dP/dX
*  for a given Gaussian latitude
*************************************************************************

      IMPLICIT REAL* 8(A-H,O-Z)

* transform resolution parameters
* grid resolution
      PARAMETER ( NLON= 128, NLAT= 64, NLOH=NLON/2, NLAH=(NLAT+1)/2 )
* spectral resolution
      PARAMETER ( MT= 42 )

      DIMENSION P(0:MT,0:MT+1), DP(0:MT,0:MT)
*------------------------------------------------------------------------
      SIA = SQRT( 1.D0 - X*X )

* m=0
      P (0,0) = 1.D0
      P (0,1) = SQRT( 3.D0 ) * X
      DP(0,0) = 0.D0

      E1 = 1.D0 / SQRT( 3.D0 )
      DO 10 N=2,MT+1
        E2 = SQRT( N*N / (4.D0*N*N - 1.D0) )
        P (0,N  ) = (X*P(0,N-1) - E1*P(0,N-2)) / E2
        DP(0,N-1) = (N-1.D0)*E2*P(0,N) - N*E1*P(0,N-2)
        E1 = E2
10    CONTINUE

* m=1,2,...,MT
      DO 20 M=1,MT
```

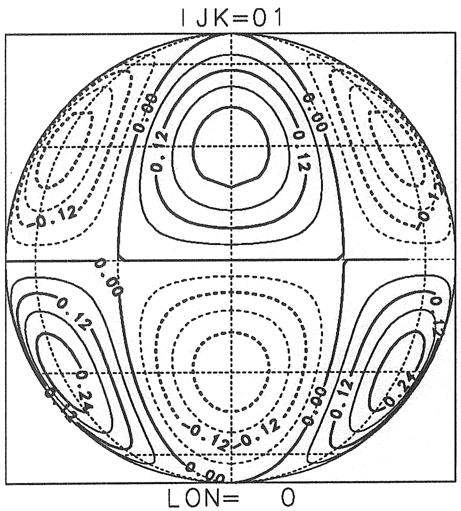
```
          P(M,M  ) = SQRT( (2.D0*M+1.D0)/(2.D0*M) ) * SIA * P(M-1,M-1)
          P(M,M+1) = SQRT(  2.D0*M+3.D0  ) * X * P(M,M)

          E1 = 1.D0 / SQRT( 2.D0*M+3.D0 )
          DP(M,M) = M*E1*P(M,M+1)

          DO 30 N=M+2,MT+1
            E2 = SQRT( (N*N - M*M) / (4.D0*N*N - 1.D0) )
            P (M,N  ) = (X*P(M,N-1) - E1*P(M,N-2)) / E2
            DP(M,N-1) = (N-1.D0)*E2*P(M,N) - N*E1*P(M,N-2)
            E1 = E2
 30       CONTINUE
 20     CONTINUE

        RETURN
        END
```

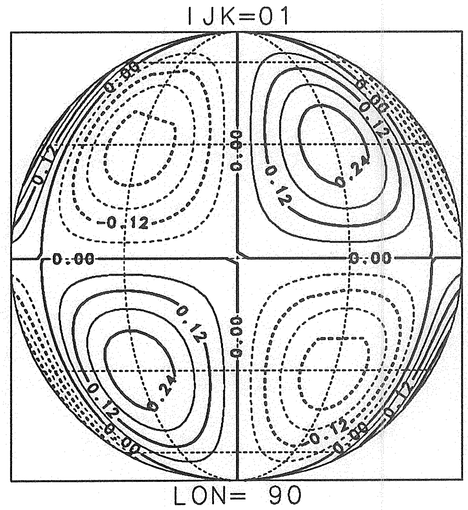

T E S T
ENERGY
TOTAL WAVENUMBER
10⁰ 10⁻¹ 10⁻² 10⁻³ 10⁻⁴ 10⁻⁵ 10⁻⁶ 10⁻⁷ 10⁻⁸
10⁰ 10¹ 10²

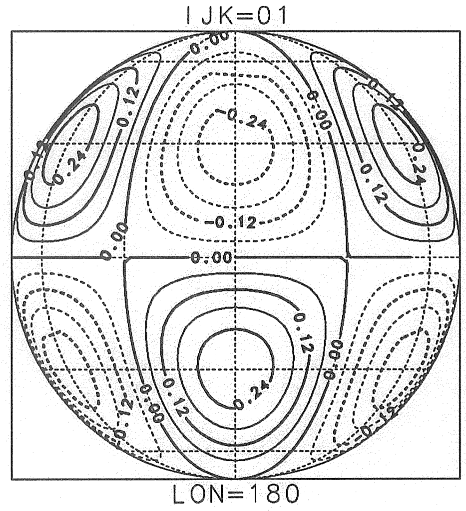

ENSTROPHY
TOTAL WAVENUMBER
10² 10¹ 10⁰ 10⁻¹ 10⁻² 10⁻³ 10⁻⁴ 10⁻⁵ 10⁻⁶
10⁰ 10¹ 10²

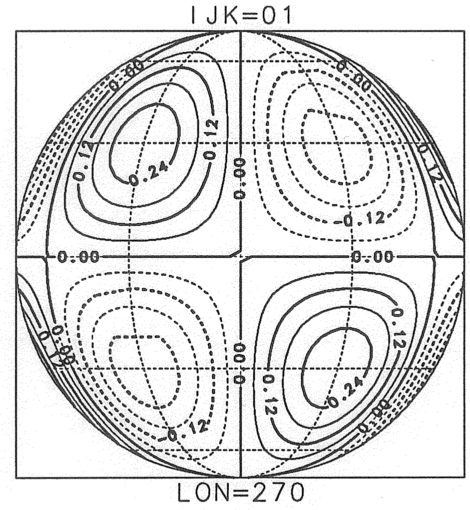

IJK=01
NORTH POLE


IJK=01
SOUTH POLE

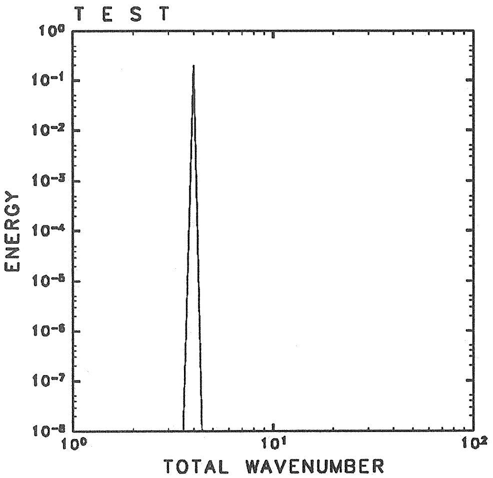

IJK=01
LON= 0

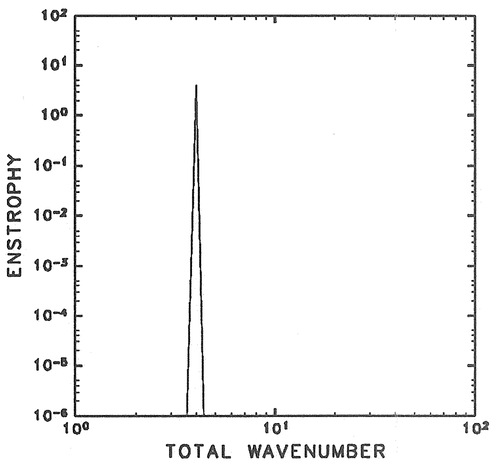

IJK=01
LON= 90

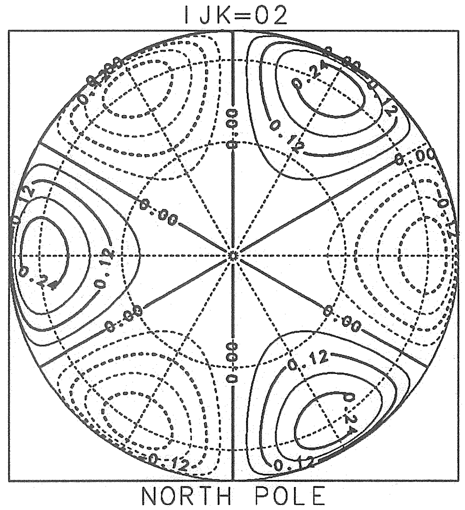

IJK=01
LON=180

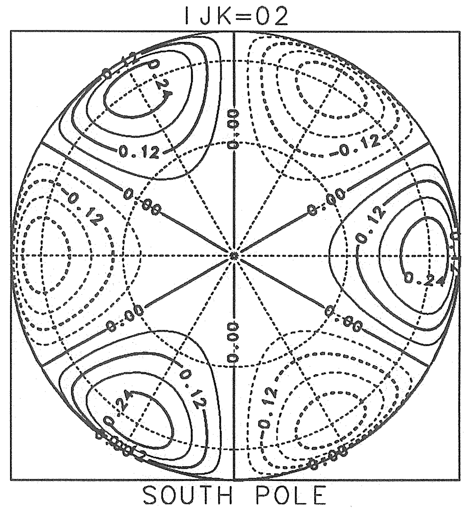

IJK=01
LON=270

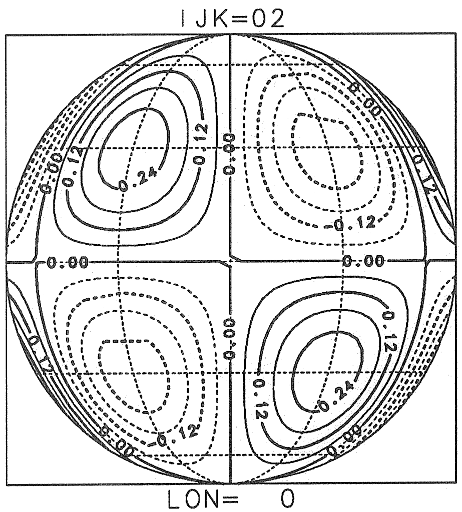
gcht.f

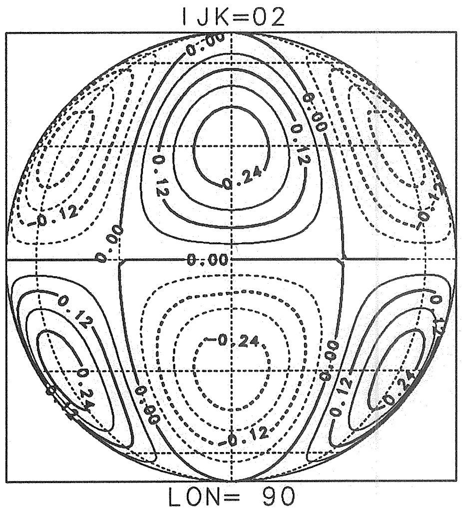

TEST
ENERGY
TOTAL WAVENUMBER

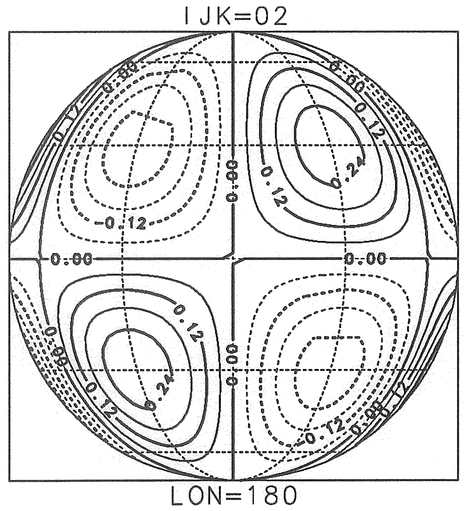

ENSTROPHY
TOTAL WAVENUMBER

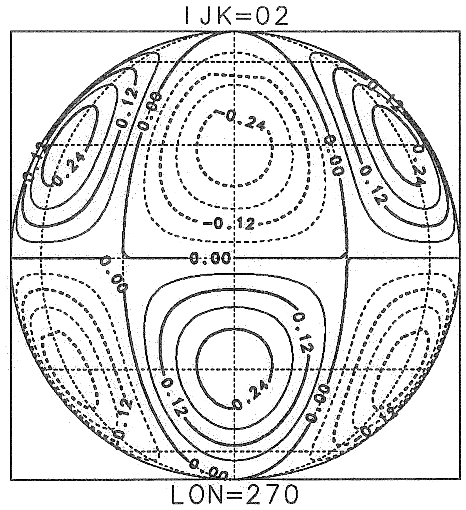

IJK=02
NORTH POLE

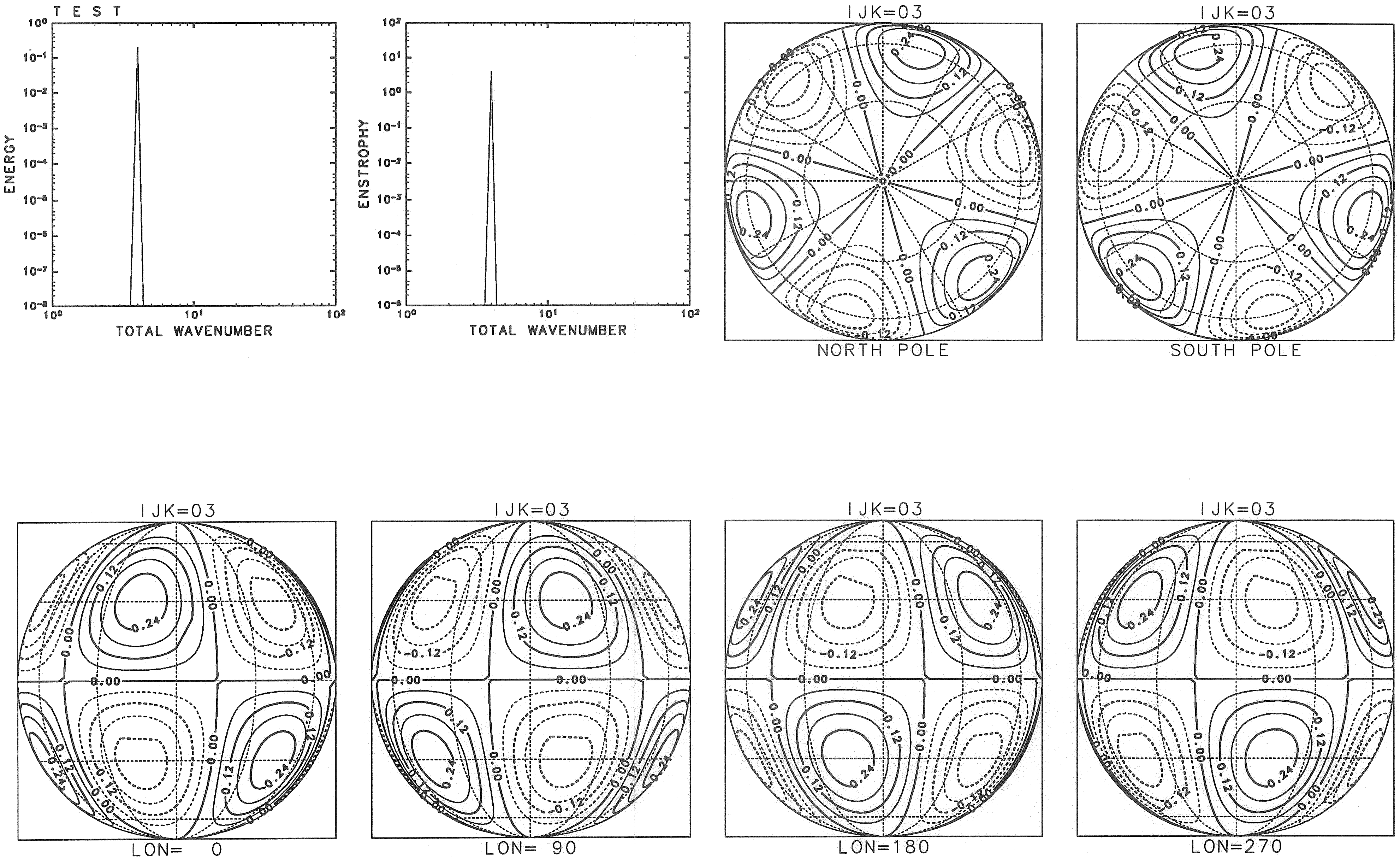

IJK=02
SOUTH POLE


IJK=02
LON= 0


IJK=02
LON= 90


IJK=02
LON=180

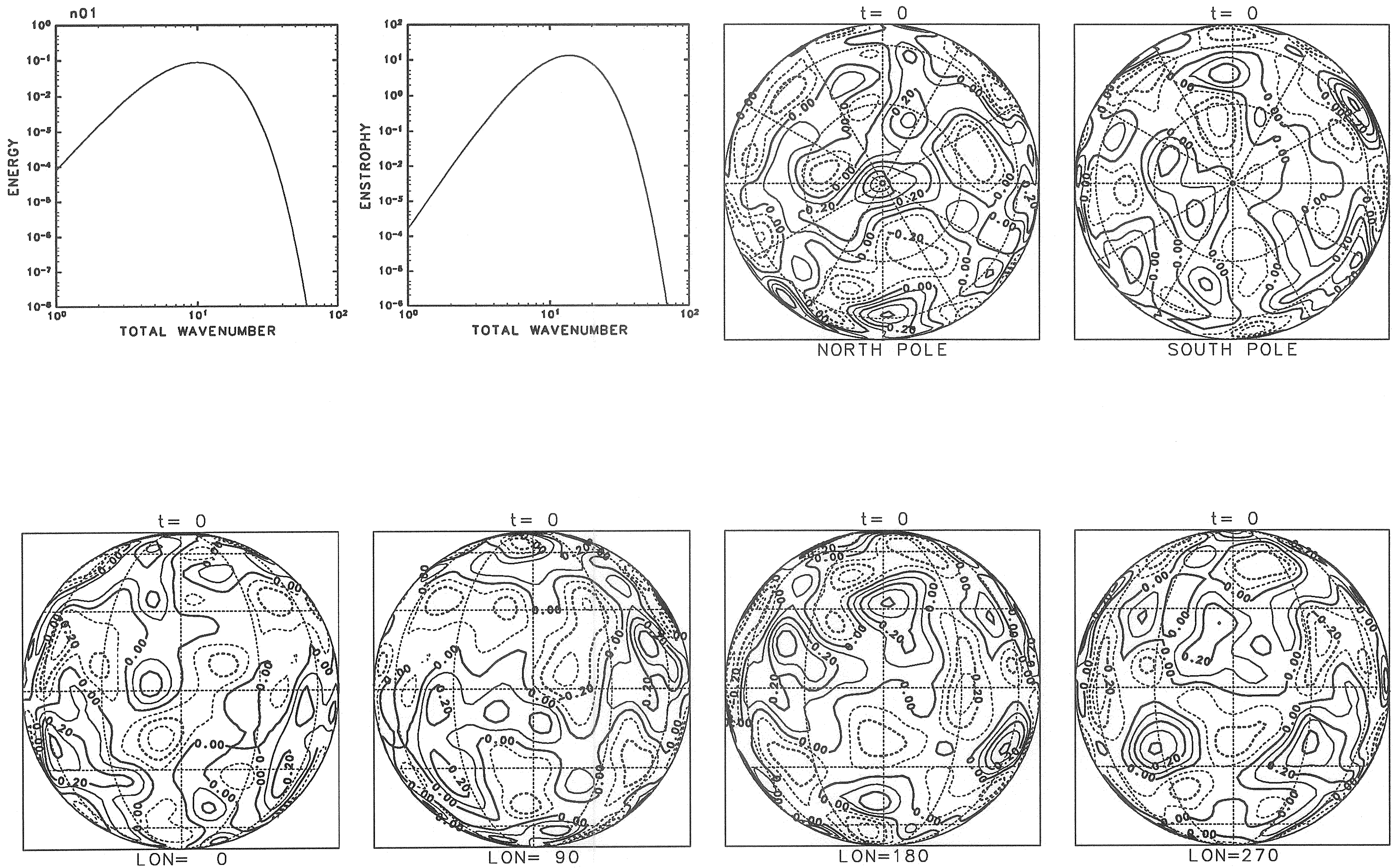

IJK=02
LON=270


TEST
ENERGY
TOTAL WAVENUMBER
ENSTROPHY
TOTAL WAVENUMBER
IJK=03
NORTH POLE
IJK=03
SOUTH POLE
IJK=03
LON= 0
IJK=03
LON= 90
IJK=03
LON=180
IJK=03
LON=270

```
***********************************************************************
*
*       set an initial value for time-integration
*           assuming an initial energy spectrum
*
***********************************************************************

      PROGRAM GINIT
      IMPLICIT COMPLEX(Z)

* transform resolution parameters
* grid resolution
      PARAMETER ( NLON= 256, NLAT= 128, NLOH=NLON/2, NLAH=(NLAT+1)/2 )
* spectral resolution
      PARAMETER ( MT= 85, MT2=(MT+1)**2 )

      DIMENSION ZP (0:MT,0:MT)
      DIMENSION ENG(0:MT), ENS(0:MT)
      DIMENSION RAM(MT2), RPH(MT2)
      DOUBLE PRECISION RAND(MT2)
      CHARACTER CWDATA*38, CW*35, CT*3

      DATA CW/'/home/stupid05/yoden/turbsp/initia/'/
*-----------------------------------------------------------------------

      PI = 4*ATAN(1.0)
      IR = 0

* parameters for initial data
      E0 = 1.0
      B  = 0.5

*-----------------------------------------------------------------------
      DO 10 IJK=1,10

* make radom numbers
        CALL URAND1( MT2, RAND, IR )
        DO 1 N=1,MT2
          RAM(N) = RAND(N)
1       CONTINUE

        CALL URAND1( MT2, RAND, IR )
        DO 2 N=1,MT2
          RPH(N) = RAND(N)
    2   CONTINUE

        SEN = 0.0
        DO 20 N=0,MT
          SEN = SEN + EXP(-B*N)*N*N*N*N*N
20      CONTINUE

        A = E0/SEN

* make initial data
        J  = 0
        K  = 0

        DO 30 N=0,MT

          ENG(N) = A*EXP(-B*N)*N*N*N*N*N

*       for m=0
          J = J + 1
          SAM = RAM(J)

          DO 40 M=1,N
```

```
            J = J + 1
            SAM = SAM + RAM(J)*2
40        CONTINUE

          DO 50 M=0,N
            K = K + 1
            PHA = RPH(K)*2*PI
            AMP = RAM(K)/SAM
            IF(N .EQ. 0) THEN
              AMP = 0
            ELSE
              AMP = SQRT( AMP * 2*ENG(N)/(N**2+N) )
            END IF

            IF(M .EQ. 0) THEN
              ZP(N,M) = AMP
            ELSE
              ZP(N,M) = AMP * EXP( (0.,1.)*PHA )
            END IF
50        CONTINUE
30      CONTINUE

* storage of the initial data on disk
        CT = 'n??'
        WRITE(CT(2:3),'(I2.2)') IJK
        CWDATA = CW // CT

        OPEN (1, FILE=CWDATA, FORM='UNFORMATTED')
        WRITE(1) MT, NLAT, ZP
        CLOSE(1)

* compute energy and enstrophy spectrum
        DO 60 N=0,MT
          ENG(N) = 0.5*N*(N+1)*ABS(ZP(N,0))**2
          ENS(N) = 0.5*N*(N+1)*ABS(ZP(N,0))**2  *N*(N+1)
        DO 60 M=1,N
          ENG(N) = ENG(N) + N*(N+1)*ABS(ZP(N,M))**2
          ENS(N) = ENS(N) + N*(N+1)*ABS(ZP(N,M))**2 *N*(N+1)
60      CONTINUE

* compute characteristic scales
        TOENG = 0.0
        TOENS = 0.0
        DO 70 N=0,MT
          TOENG = TOENG + ENG(N)
          TOENS = TOENS + ENS(N)
70      CONTINUE

        CVEL = SQRT( TOENG )
        CLEN = SQRT( TOENG/TOENS )
        CTIM = CLEN/CVEL

* output
        WRITE(6,600) IJK, A, B
600     FORMAT(2X, '<<< INITIAL STATE >>>'/
     +         5X, 'J =', I2.2, 5x, 'A =', E12.5, 5X, 'B =', F5.2)
        WRITE(6,610) TOENG, TOENS, CVEL, CLEN
610     FORMAT(5X, 'TOTAL ENERGY =', E12.5,
     +         5X, 'TOTAL ENSTROPHY =', E12.5,/
     +         5X, 'VELOCITY SCALE =', E12.5, /
     +         5X, 'LENGTH SCALE   =', E12.5, /)
10     CONTINUE
*-----------------------------------------------------------------------------

       STOP
       END
```

```
************************************************************************
      SUBROUTINE URAND1(N, X, IR)
************************************************************************
* UNIFORM RANDOM NUMBER GENERATOR (MIXED CONGRUENTIAL METHOD)          *
*     PORTABLE BUT SLOW.  THE PERIOD IS ONLY 1664501.                  *
* PARAMETERS                                                           *
*   (1) N      (I) THE NUMBER OF RANDOM NUMBERS TO BE GENERATED        *
*                  (INPUT)                                             *
*   (2) X      (D) UNIFORM RANDOM NUMBERS (OUTPUT)                     *
*   (3) IR     (I) THE INITIAL SEED  (INPUT)                           *
*                  THE SEED FOR THE NEXT CALL (OUTPUT)                 *
* COPYRIGHT: Y. OYANAGI, JUNE 30, 1989  V.1                            *
************************************************************************
*
       DOUBLE PRECISION X(N), INVM
       PARAMETER (M = 1664501, LAMBDA = 1229, MU = 351750)
       PARAMETER (INVM = 1.0D0 / M)
*PAREMETER CHECK
      IF( N .LE. 0) THEN
       WRITE(6,*) '(SUBR.URAND1) PARAMETER ERROR. N = ', N
       WRITE(6,*) 'RETURN WITH NO FURTHER CALCULATION.'
       RETURN
      END IF
      IF( IR .LT. 0 .OR. IR .GE. M) THEN
       WRITE(6,*) '(SUBR.URAND1) WARNING. IR = ', IR
      END IF
*MAIN LOOP
      DO 10 I = 1, N
       IR = MOD( LAMBDA * IR + MU, M)
       X(I) = IR * INVM
   10 CONTINUE
      RETURN
      END
```


n01
ENERGY
TOTAL WAVENUMBER
ENSTROPHY
TOTAL WAVENUMBER
t= 0
NORTH POLE
t= 0
SOUTH POLE
t= 0
LON= 0
t= 0
LON= 90
t= 0
LON=180
t= 0
LON=270

NO.

reference (ψ) と difference ($\delta\psi$)

$$\omega = \Delta\psi$$

$$\delta\omega = \Delta\delta\psi$$

full eq.

$$\frac{\partial\omega}{\partial t} + J(\psi, \omega) \cdots$$

linear tangential eq.

$$\frac{\partial\delta\omega}{\partial t} + J(\delta\psi, \omega) + J(\psi, \delta\omega) \cdots$$

? i) initial condition for $\delta\psi$ ← (ψと同じスペクトル 別の乱数)

ii) renormalization

$\delta\psi$ エネルギー . 元のエネルギーの100倍以上

↓

$\frac{1}{100}$.

(1) 何回 renormalize (2か)

NO.

Jacobian の計算：

$$J(\psi, \omega) = \frac{\partial \psi}{\partial \lambda}\frac{\partial \omega}{\partial \mu} - \frac{\partial \psi}{\partial \mu}\frac{\partial \omega}{\partial \lambda}$$

$$= \frac{\partial}{\partial \mu}\left(\frac{\partial \psi}{\partial \lambda}\omega\right) - \frac{\partial^2 \psi}{\partial \mu \partial \lambda}\omega - \frac{\partial}{\partial \lambda}\left(\frac{\partial \psi}{\partial \mu}\omega\right) +$$

計算1 a weak nonlinear system

NO.

支配方程式

wave : (2-10)

$$\frac{\partial \zeta'}{\partial t} + \frac{1}{a^2} J(\bar{\psi}, \zeta') + \frac{1}{a^2} J(\psi', \bar{q}) = F' \qquad (C-1)$$

zonal : (2-15)

$$\frac{\partial \bar{\zeta}}{\partial t} + \frac{1}{a^2 \cos\phi} \frac{\partial}{\partial \phi} \left(\overline{\zeta' \frac{\partial \psi'}{\partial \lambda}} \right) = \bar{F} \qquad (C-2)$$

$$\zeta' = \Delta_2 \psi'$$

$$\bar{\zeta} = \frac{1}{a^2 \cos\phi} \frac{\partial}{\partial \phi} \left(\cos\phi \frac{\partial \bar{\psi}}{\partial \phi} \right)$$

$$\bar{q} = \bar{\zeta} + 2\Omega \sin\phi$$