Climate Change: some basic physical concepts and simple models

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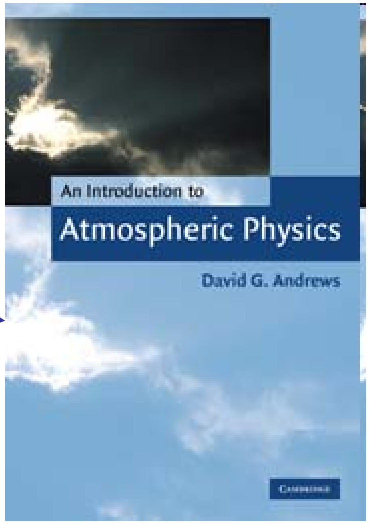
Some of you have used my textbook 'An Introduction to Atmospheric Physics' (IAP)



I am now preparing a 2nd edition.

The main difference will be a new chapter on Climate Physics.

This lecture will cover some of the new material.

Outline

- The physics of climate change is a very complex subject!
- However, many of the most important ideas can be described using very simple models. ('Toy models'.)
- An example is the Energy Balance Model (EBM).

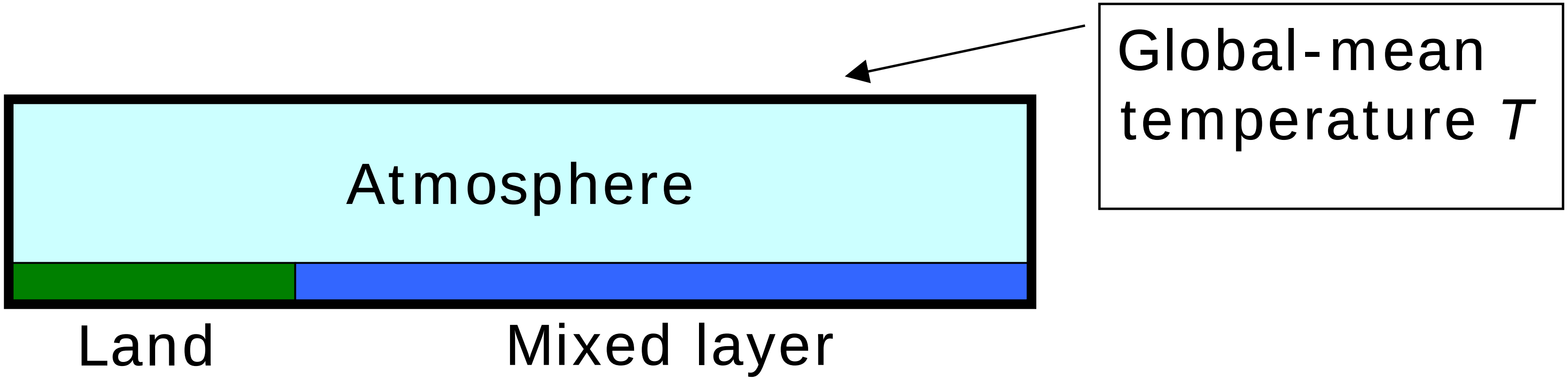
I shall focus on:

- Response of EBM to radiative forcing
  - Several examples
  - CO_2 stabilisation
  - Clarification of Andrews and Allen figure from previous lecture

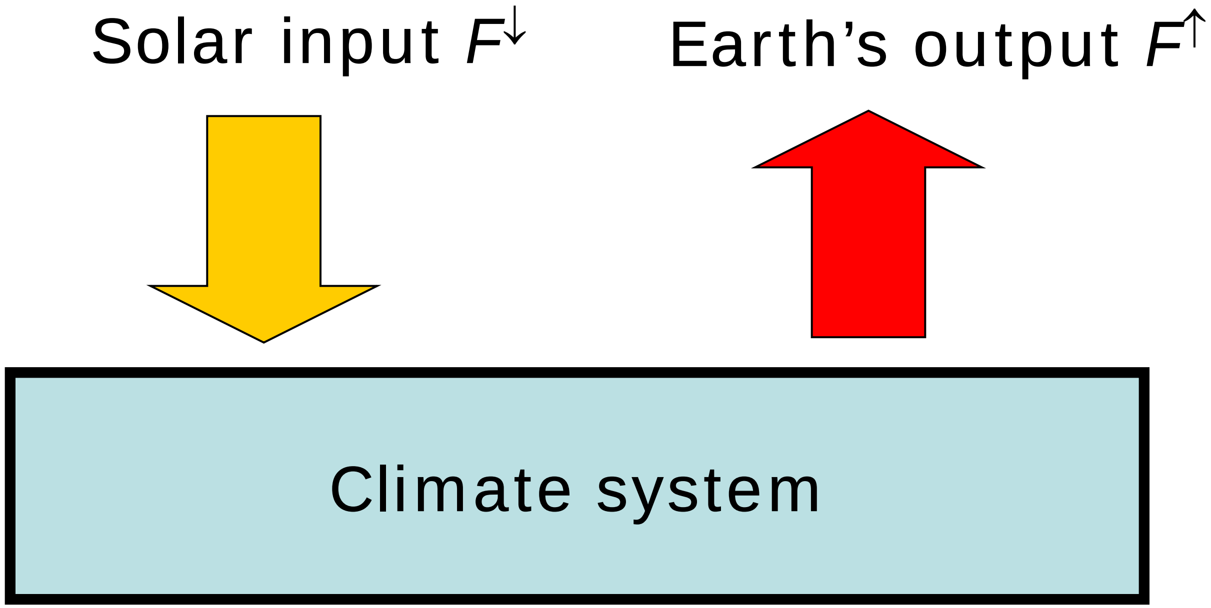
- Introduction to climate feedbacks
  [if there is time]

Energy Balance Model

- Simplest case: the 'climate system' includes the atmosphere, land surface and 'mixed layer' (= top 100m of ocean), but not deep ocean:



Radiative power input and output (per unit area)



- Solar input $F^{\downarrow}$: 'short-wave' radiation (visible, ultra-violet), wavelength $\leq 4\ \mu$m
- Earth's output $F^{\uparrow}$: 'long-wave' radiation (infra-red), also called 'thermal' radiation, wavelength $\geq 4\ \mu$m.

In equilibrium:

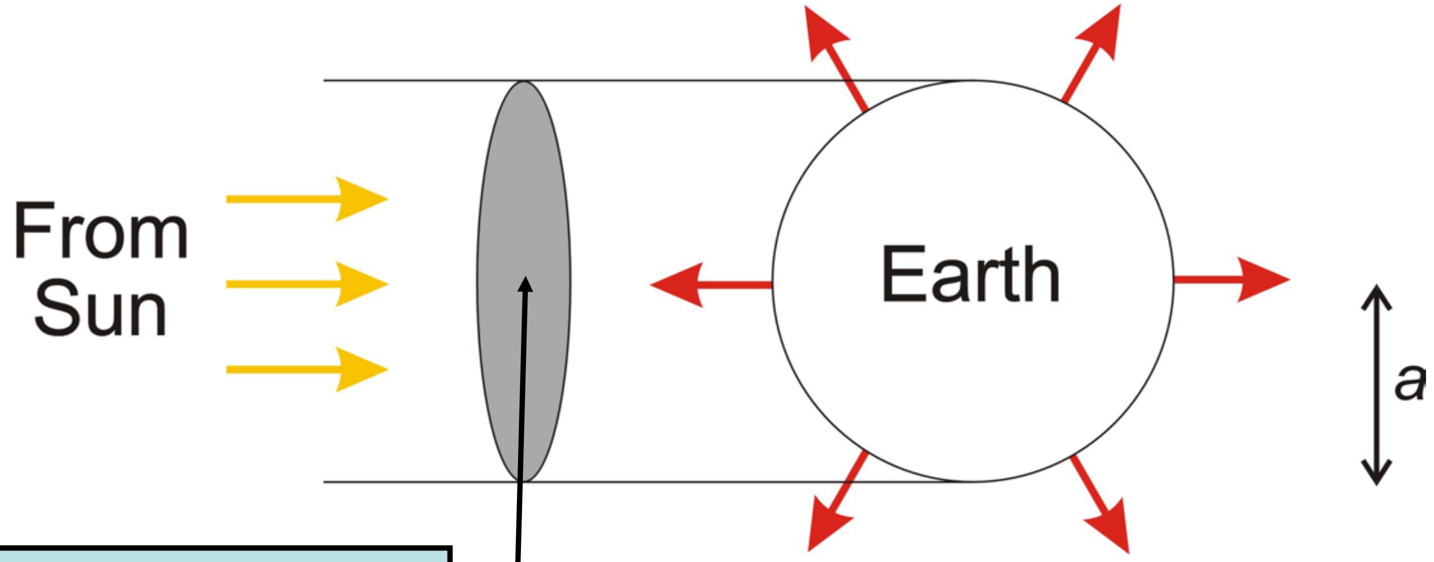
Solar input = Earth’s output

Out of equilibrium:

Solar input ≠ Earth’s output

⇒ Climate change

Equilibrium climate: simple models



~ 1370 W m^{-2} short-wave radiation from Sun, of which ~ 30% is reflected

[See IAP, 1.3.1]

- Earth intercepts solar beam over its cross-sectional area, πa^2
- So incident power = 1370 x 0.7 *x* πa^2 Watts (short wave).
- But it emits (long-wave) power from *all* its surface, $4\pi a^2$.

- *Assume* Earth acts like a **black body** at (absolute) temperature T, so that it emits **power σT^4 per unit area**, where

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

is the Stefan-Boltzmann constant.

So in equilibrium,

incoming power = outgoing power:

$1370 \times 0.7 \times \pi a^2 = 4\pi a^2 \times \sigma T^4$.

Cancel πa^2 and rearrange:

$T = \left(\frac{1370 \times 0.7}{4\sigma}\right)^{1/4} \approx 255\,\mathrm{K}$.

[Note for later use: $F^{\uparrow} = \sigma T^4 \approx 240\,\mathrm{W\,m^{-2}}$.]

- This temperature (255 K) is much lower than typical observed global-mean T ~ 290 K!

- So this model misses some important physics, especially the effects of *greenhouse gases.*

Greenhouse gases

- Are gases that absorb and emit infra-red radiation but allow solar radiation to pass through without significant absorption. They affect $F^{\uparrow}$ but not $F^{\downarrow}$.
- Examples: Water vapour (H_2O) and carbon dioxide (CO_2) are the two most important in determining the *current* climate.
- Increasing CO_2 is a major contributor to climate change.
- Effect of H_2O in climate change is more complex.

- Very simple model of effect of greenhouse gases in producing current climate: see IAP, 1.3.2.

- In this lecture I shall mostly focus on climate change.

Consider unit area of climate system

Let heat capacity per unit area $= C$ (mostly in mixed layer). Then

$$C\frac{dT}{dt} = F^{\downarrow} - F^{\uparrow} \,.$$

Assume net heat flux into system

$$F^{\downarrow} - F^{\uparrow} = Q(T,U)$$

where $T =$ global-mean temperature and $U =$ concentration of some greenhouse gas.

Steady-state (equilibrium) climate

In a steady-state climate at constant temperature $T = T_0$ and concentration $U = U_0$, we have $CdT/dt = 0$

$$\Rightarrow \quad Q(T_0, U_0) = 0$$

i.e. net heat flux into system is zero.

Perturbed climate

Perturb steady state, so

$$T = T_0 + T'(t), \qquad U = U_0 + U'(t),$$

where T' and U' are small, $\Rightarrow$

$$\begin{aligned} C\frac{dT'}{dt} &= Q(T_0 + T', U_0 + U') \\ &\approx \frac{\partial Q}{\partial T}T' + \frac{\partial Q}{\partial U}U' \end{aligned}$$

Taylor expansion

Radiative forcing

Define *radiative forcing* by

$$F = \frac{\partial Q}{\partial U} U'$$

Example: *U*' might be an increase in CO_2 concentration

Climate feedback

Define by $\lambda = -\frac{\partial Q}{\partial T}$

Examples:

• 'Black body feedback': warmer Earth emits more long-wave radiation,

$\lambda > 0$, negative feedback

• Water-vapour feedback: warmer atmosphere contains more water vapour, traps more long-wave radiation,

$\lambda < 0$, positive feedback.

Equation for EBM

So we get

$$C\frac{dT'}{dt} + \lambda T' = F(t)$$

Radiative forcing

Heat capacity

Rate of change of heat content of climate system

Feedback

Solution of EBM for some specified time-dependent radiative forcings *F(t)*

$$C\frac{dT'}{dt} + \lambda T' = F(t) \qquad (*)$$

F(t) could represent, say, radiative forcing due to an increase of greenhouse gases such as CO_2.

General solution

Define *feedback response time*

$$\tau = C/\lambda \, .$$

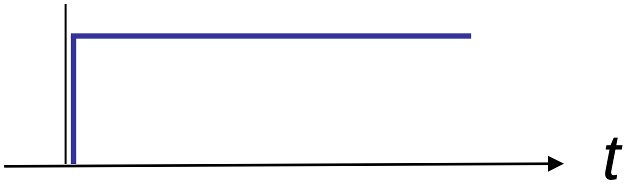
To solve equation $(*)$, divide by C and multiply by the integrating factor exp(t/τ) to get

$$\frac{d}{dt}\left(T' e^{t/\tau}\right) = \frac{F(t)}{C}\, e^{t/\tau} \, .$$

Assume $T' = 0$ at an initial time $t = 0$, and integrate $(*)$ to obtain formal solution

$$T'(t) = \frac{e^{-t/\tau}}{C} \int_0^t F(u)\, e^{u/\tau}\, du \, .$$

Some examples of *F(t)*



(a) Step-function forcing

$$F(t) = 0 \quad \text{for} \quad t \leq 0\,, \qquad = F_1 \quad \text{for} \quad t > 0\,,$$

where F_1 is a constant. Then temperature response is given by

$$T'(t) = S\left(1 - e^{-t/\tau}\right) \quad \text{for} \quad t > 0\,,$$

where

$$S = \frac{F_1}{\lambda}$$

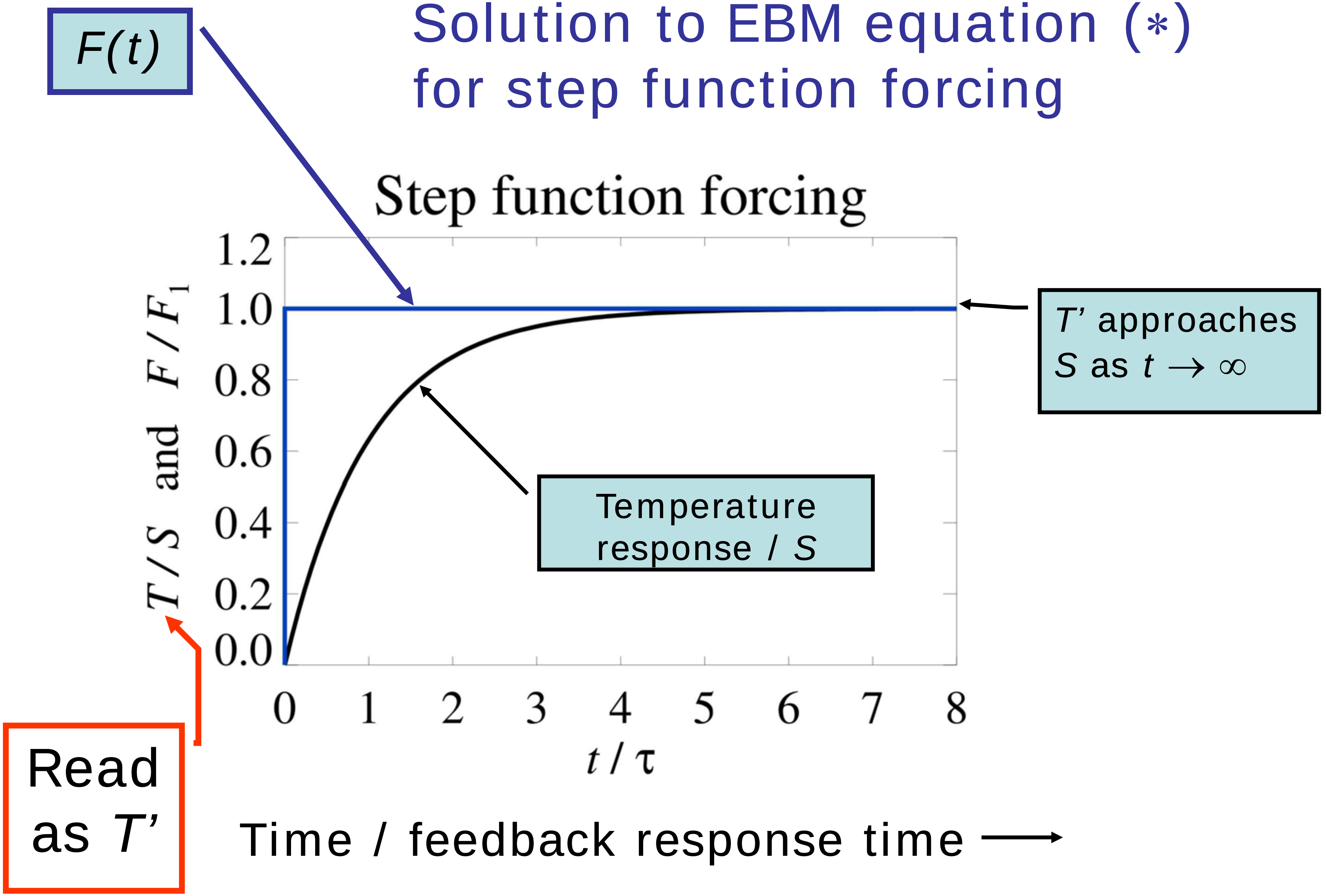
is called the *equilibrium climate sensitivity* .

Equilibrium climate sensitivity

The equilibrium climate sensitivity S is the solution to EBM equation $(*)$ when CdT'/dt is negligible,
i.e. long-term steady-state solution:

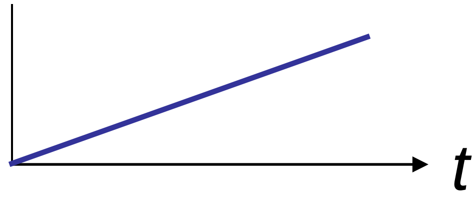
$$T' = \boxed{S \equiv \frac{F_1}{\lambda}\,.}$$

Solution to EBM equation (*) for step function forcing



Time / feedback response time ⟶

(b) Ramp forcing



Radiative forcing increases linearly with time:

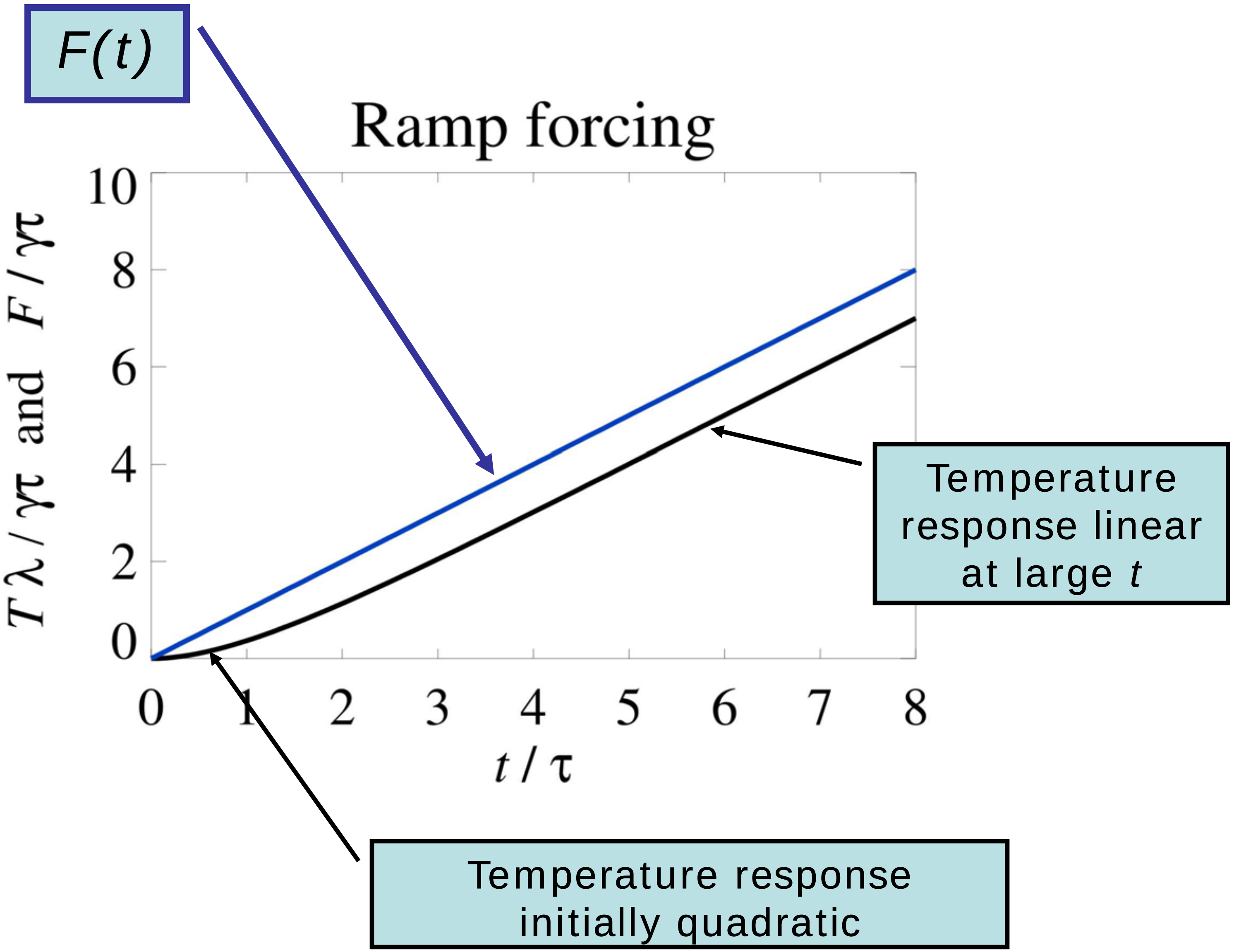
$$F(t) = 0 \quad \text{for} \quad t \leq 0 \,, \qquad = \gamma t \quad \text{for} \quad t > 0 \,,$$

where γ is constant. Good representation of the radiative forcing due to increasing CO_2.

Solution is

$$T'(t) = \frac{\gamma\tau}{\lambda}\left(\frac{t}{\tau} - 1 + e^{-t/\tau}\right) \quad \text{for} \quad t > 0 \,.$$

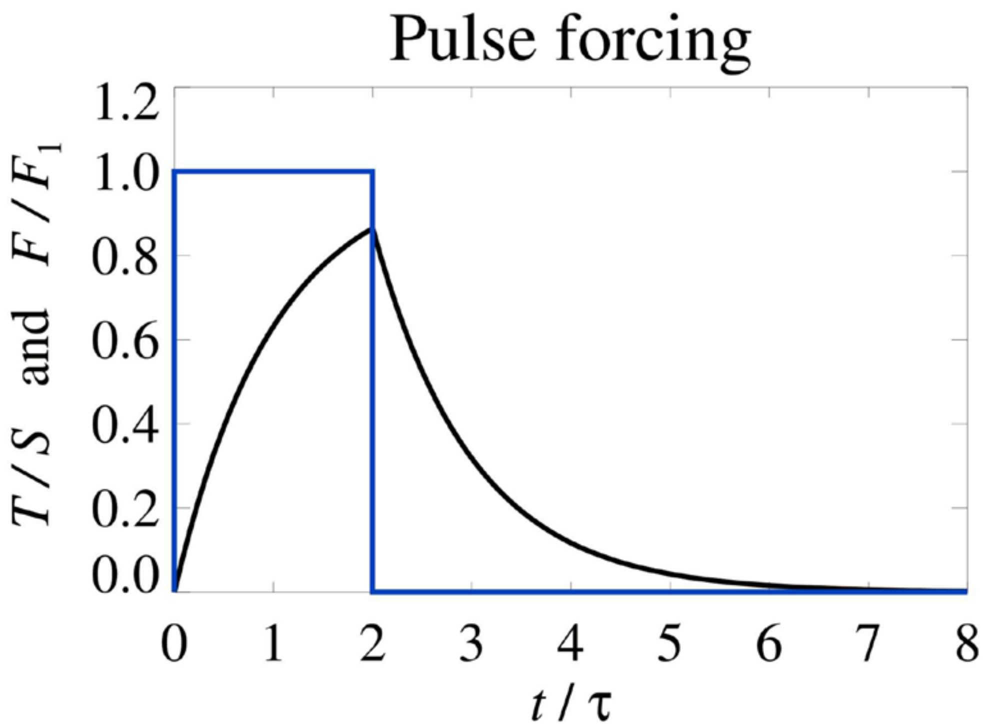
(Exercise for student!)


F(t)
Ramp forcing
T λ / γτ and F / γτ
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
t / τ
Temperature response linear at large t
Temperature response initially quadratic

(c) Pulse forcing

$$F(t) = 0 \quad \text{for} \quad t \leq 0 \quad \text{and} \quad t \geq t_0 ,$$
$$= F_1 \quad \text{for} \quad 0 < t < t_0 .$$

Example: massive volcanic eruption ($F_1 < 0$).



(d) Sinusoidal forcing

$$F(t) = F_2 \cos(\omega t)$$

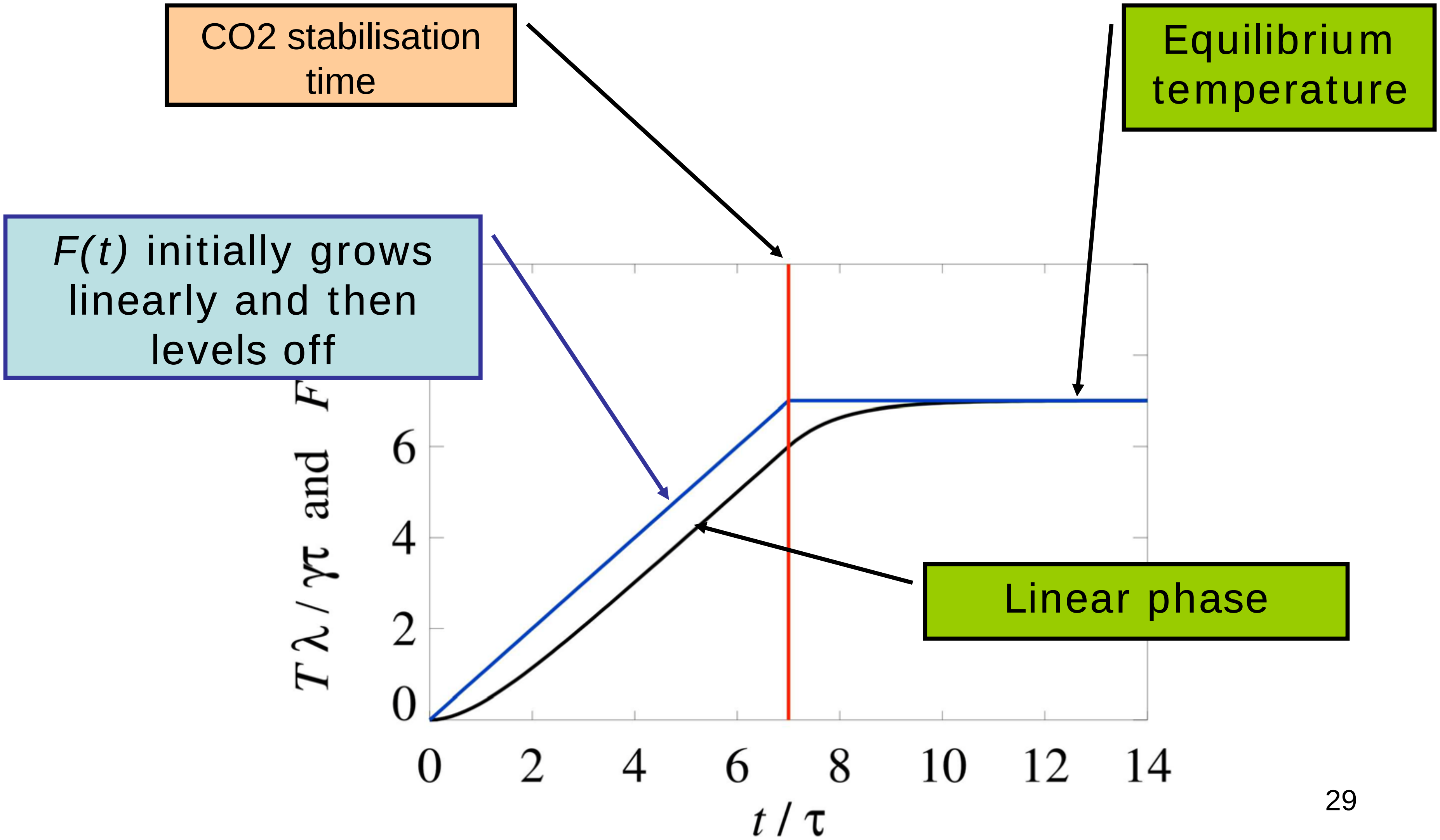
for all t, where F_2 is constant; seek a purely oscillatory response.

Example: forcing by the 11-year solar cycle.

(This is similar to an AC electric circuit calculation.)

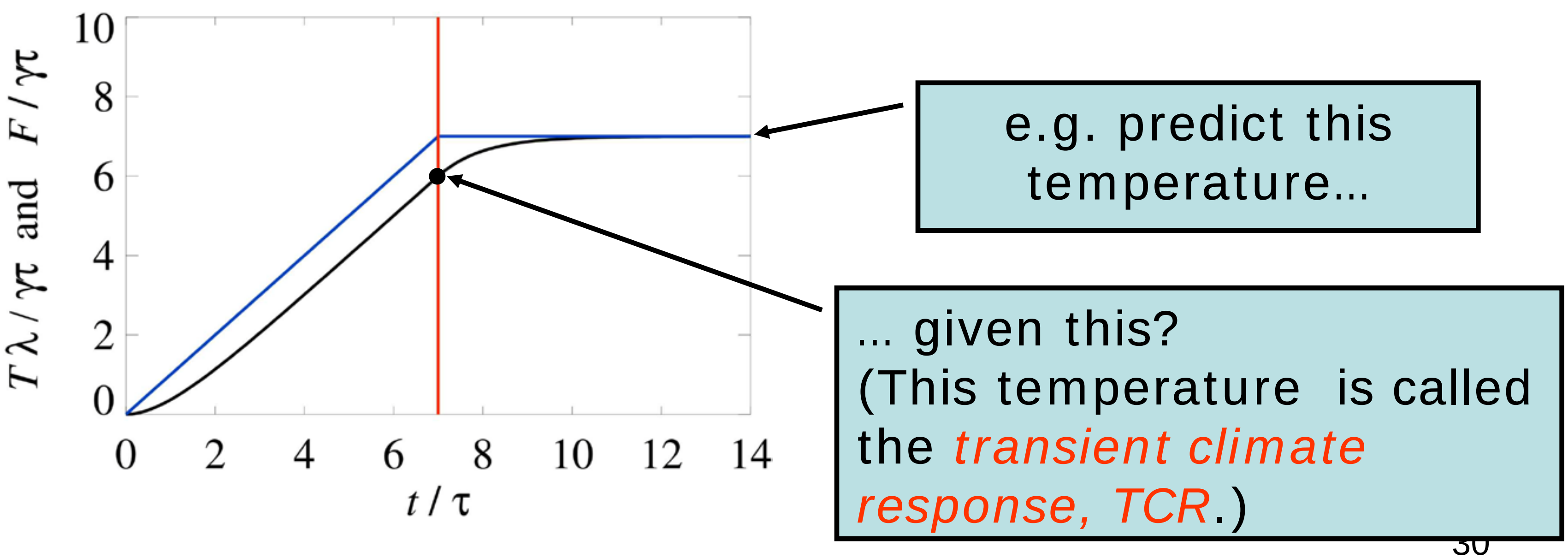
(e) Ramp followed by steady forcing

Relevant to "CO_2 stabilisation" scenario?



What can this model tell us about how the climate might respond to CO_2 stabilisation?

- Can we forecast the equilibrium temperature, given information at some earlier time?



- This is easy if
  - we have a perfect model
  - we know all the parameters.

- But it is difficult to do in practice!

- Even if we believe our EBM represents the physics quite well (???), we still have to know the values of parameters C and λ.

A possible approach

- Forecast (say) 100 years ahead with very complex general circulation model.
- Fit results for global-mean temperature to EBM over this 100 years.
- Then use EBM for forecast beyond 100 years.

- But there will be uncertainties in the fitted values of *C* and λ.

(Note: the value of the heat capacity *C* can’t be determined “from first principles”, because of likely importance of the deep ocean.)

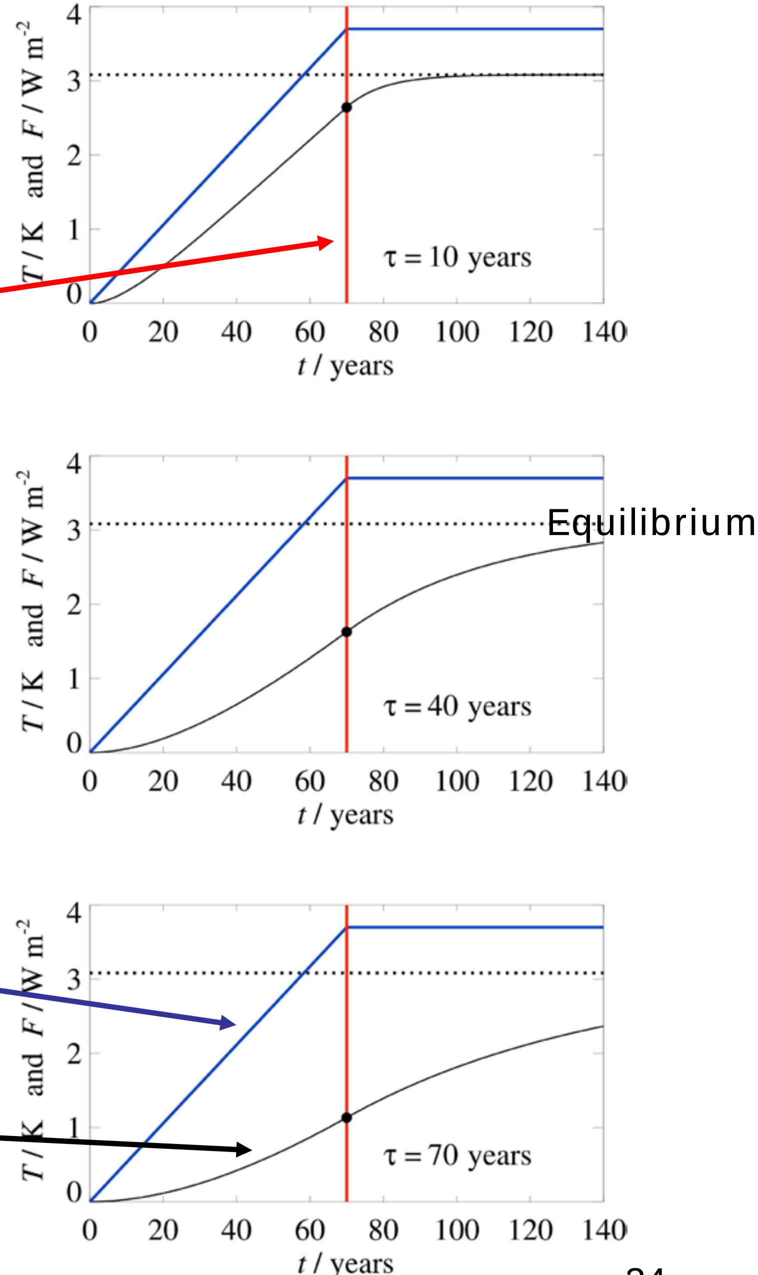
- This may make it difficult to get an accurate value for equilibrium temperature.

Example

Take ramp time = 70 years.

Fix λ = 1.2 Wm^{-2} K^{-1}

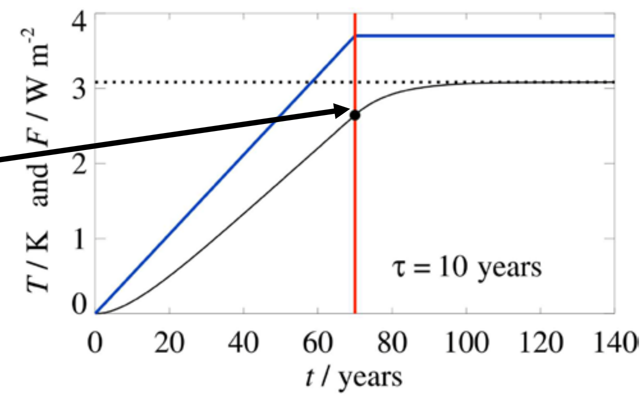
Vary τ = C/ λ: 10 years, 40 years, 70 years.



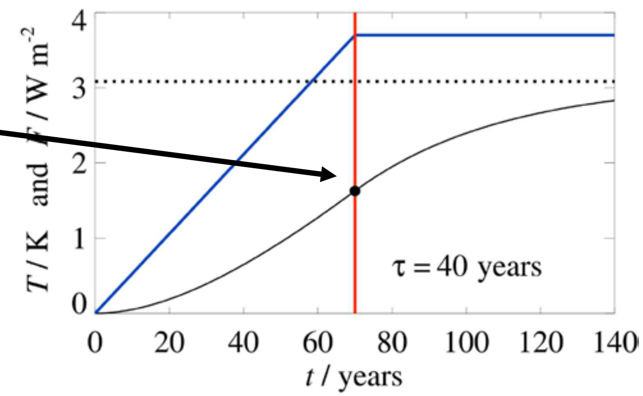
Forcing

Temperature response

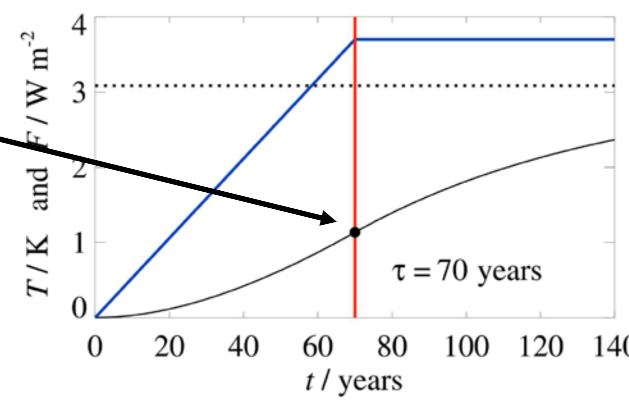
τ = 10 years: 70-year *T'* is 88% of equilibrium value



τ = 40 years: 70-year *T'* is 54% of equilibrium value



τ = 70 years: 70-year *T'* is 38% of equilibrium value



- So given the temperature at 70 years, the equilibrium value depends strongly on the (poorly-known) value of C.

- But actually, the value of the feedback parameter λ is even more poorly known!

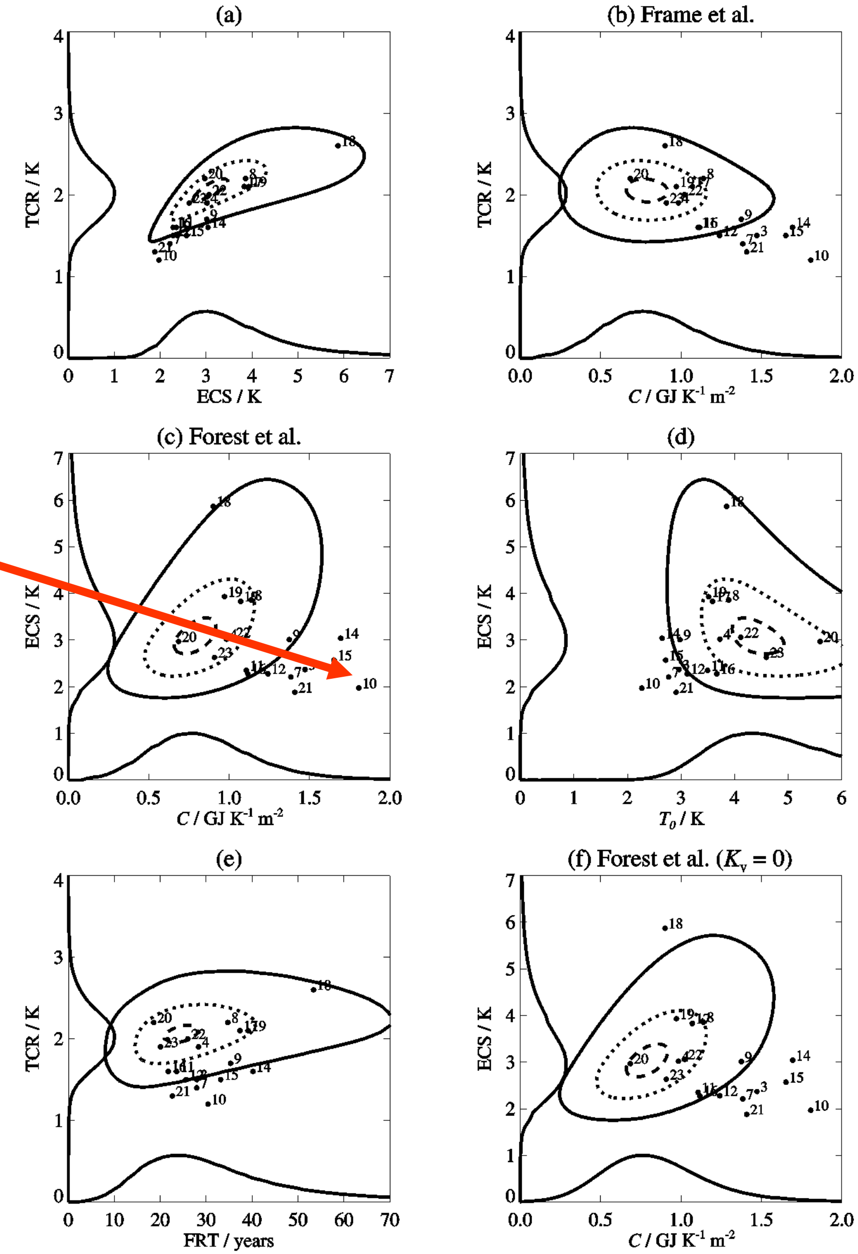
Use of EBM to provide diagnostics for IPCC climate forecasting models

(Andrews & Allen, *Atmos. Sci. Lett.* **9**, 7-12, 2008)

Each GCM run (numbered dot) gives an estimate of parameters such as:

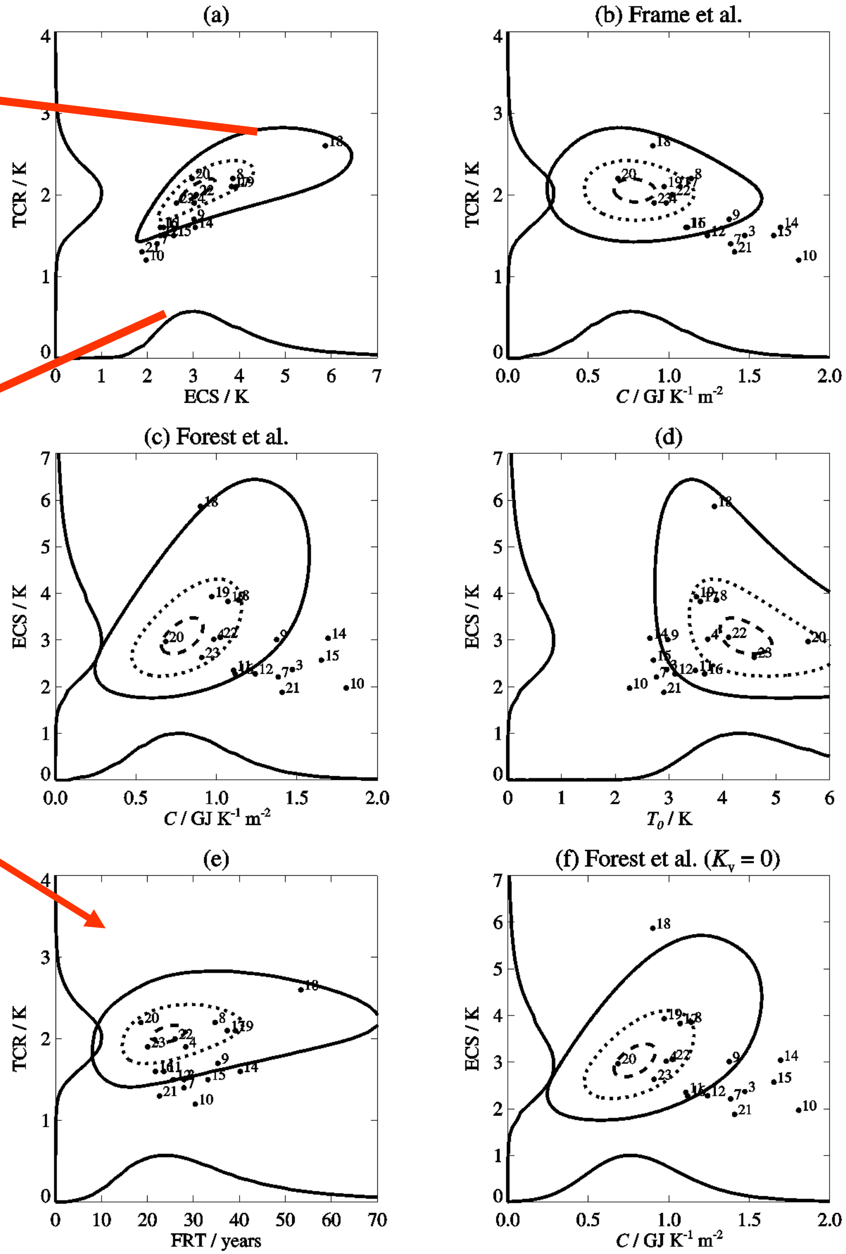
TCR,
ECS = S,
Heat capacity = C,
Feedback response time = τ

These are not all independent of each other.



Contours: likelihood calculated for an observationally-constrained ensemble of 28,800 simple climate models

Projection of likelihood onto axis



Clearest interpretation of GCM results is in terms of ***feedback response time*** and ***transient climate response***.

The GCMs systematically **underestimate** the TCR compared with observations

Climate Feedbacks

Earlier I said: assume net heat flux into system

$$F^{\downarrow} - F^{\uparrow} = Q(T, U)$$

where T = global-mean temperature and
U = concentration of some greenhouse gas

and climate feedback parameter

$$\lambda = -\frac{\partial Q}{\partial T} \, .$$

Simplest case: black body feedback

Assume 'climate system' behaves like a black body: $F^{\uparrow} = \sigma T^4$ where σ is the Stefan-Boltzmann constant.

This would be true, with $T = 255\,\mathrm{K}$, if there were no greenhouses gases.

Assume $F^{\downarrow} \approx 240\,\mathrm{W\,m^{-2}}$, independent of T.

So $\lambda_{\mathrm{BB}} = -\frac{\partial Q}{\partial T} = \frac{\partial F^{\uparrow}}{\partial T} = 4\sigma T^3 \approx 3.8\,\mathrm{W\,m^{-2}\,K^{-1}}$.

This is > 0: warmer planet radiates more, giving *negative feedback*.

Water vapour feedback

Take $Q = Q(T,\ V(T))$, where $V(T)$ is saturation mixing ratio at the mean surface pressure.

The corresponding feedback parameter is

$$\lambda_{\text{wv}} = -\frac{\partial Q}{\partial V}\frac{dV}{dT}\ .$$

By Clausius-Clapeyron equation,

$$\frac{dV}{dT} = \frac{VL}{R_{\text{v}}T^2} > 0$$

But $\partial Q/\partial V > 0$, since $F^{\uparrow}$ decreases as water vapour amount increases, $\Rightarrow \lambda_{\text{wv}} < 0$.

Calculations show that $\lambda_{\text{total}} = \lambda_{\text{BB}} + \lambda_{\text{wv}} > 0$, for current Earth, so overall still get negative feedback $\Rightarrow$ stable climate.

However, if conditions gave a large enough $-\lambda_{\text{wv}}$, we could get $\lambda_{\text{total}} < 0$, $\Rightarrow$ positive feedback: the *runaway greenhouse effect*.

Did this happen on Venus? Could it happen on Earth??? (Probably only if temperature gets *much* hotter.)

The End