

# Surface zonal flows induced by thermal convection in a rapidly rotating thin spherical shell

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# Outline

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Background

Formulation and experiments setup

Results

Summary

# Outline

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Background

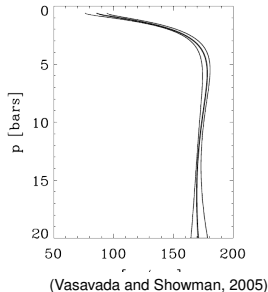
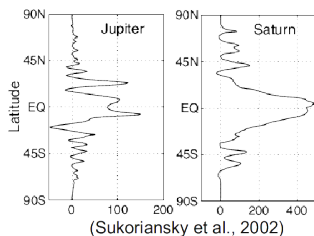
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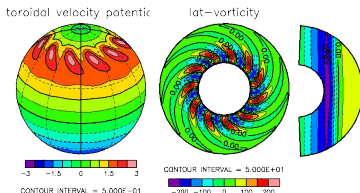
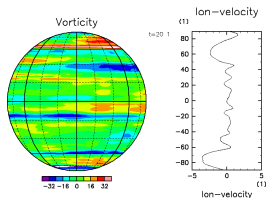
# Surface flows of gas giant planets

- Surface flows of Jupiter and Saturn are characterized by the broad prograde jets around the equator and the narrow alternating jets in mid- and high-latitudes.
- It is not yet clear whether those surface jets are produced by convective motions in the “deep” region, or are the result of fluid motions in the “shallow” weather layer.



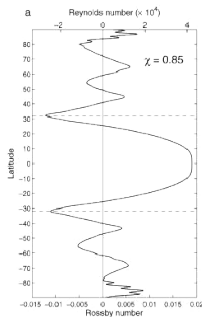
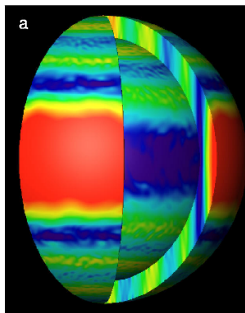
# “Deep” models and “Shallow” models

- “Shallow” models
  - 2D turbulence on a rotating sphere
  - Primitive model
    - Result: Narrow alternating jets in mid- and high-latitudes.
    - Problem: the equatorial jets are not necessarily prograde
- “Deep” models
  - Convection in rotating spherical shells
    - Result: Produce equatorial prograde flows easily
    - Problem: difficult to generate alternating jets in mid- and high-latitudes



# “Thin” spherical shell model

- Heimpel and Aurnou(2007) (hereafter, HA2007)
  - “Thin” spherical shell model with large Rayleigh number, small Ekman number.
  - Prograde jets and alternating jets in mid- and high-latitudes can produce simultaneously
  - However, **eight-fold symmetry** in the longitudinal direction is assumed.



# Purpose

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- HA2007: eight-fold symmetry in the longitudinal direction is assumed.
  - The artificial limitation of the computational domain may influence on the structure of the global flow field.
    - Zonal flows may not develop efficiently due to the insufficient upward cascade of two-dimensional turbulence
    - Stability of mean zonal flows may change with the domain size in the longitudinal direction.
    -
- In the present study:
  - Numerical simulations in the **whole** thin spherical shell domain.
  - Coarse spatial resolution and slow rotation rate are used due to the limit of computational resources.

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# Model setup

- Boussinesq fluid in a rotating spherical shell.
  - scaling: the shell thickness, viscous diffusion time, temperature difference.

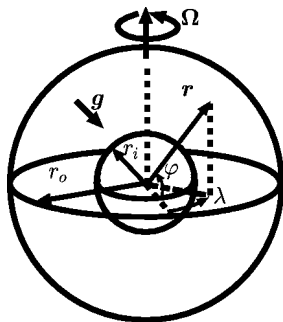
$$\nabla \cdot \mathbf{u} = 0,$$

$$E \left\{ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla^2 \mathbf{u} \right\} + 2\mathbf{k} \times \mathbf{u} + \nabla p = \frac{\text{Ra} E^2}{\text{Pr}} \frac{\mathbf{r}}{r_o} T,$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \frac{1}{\text{Pr}} \nabla^2 T.$$

- Parameters:

- Prandtl number:  $\text{Pr} = \frac{\nu}{\kappa}$
- Rayleigh number:  $\text{Ra} = \frac{\alpha g_o \Delta T D^3}{\kappa \nu}$
- Ekman number:  $E = \frac{\nu}{\Omega D^2}$
- radius ratio:  $\eta = \frac{r_i}{r_o}$



## Experimental setup

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- Boundary condition: Isothermal, Impermeable and Stress free.
- Input parameters:

parameters	present study	HA2007
Prandtl number: Pr	0.1	0.1
Radius ratio: $\eta$	0.75	0.85, 0.9
Ekman number: E	$10^{-4}$	$10^{-6}$
Modified Rayleigh number: $Ra^*$	0.05, 0.1	0.05

- the definition of modified Rayleigh number:  $Ra^* = \frac{RaE^2}{Pr} = \frac{\alpha g \Delta T D}{\Omega^2 D}$ 
  - the ratio of Coriolis term and buoyancy term
- Output parameters:
  - (local) Reynolds number,  $Re$ , is equivalent to the non-dimensional velocity in the chosen scaling.
  - (local) Rossby number:  $Ro = ERe$

# Numerical methods

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- Traditional spectral method.
  - Toroidal and Poloidal potentials of velocity are introduced.
  - The total wave number of spherical harmonics is truncated at 170, and the Chebychev polynomials are calculated up to the 48th degree.
    - The numbers of grid points:512, 256, and 48 in the longitudinal, latitudinal, and radial directions, respectively.
- In order to save computational resources, we use hyperdiffusion with the same functional form as the previous studies

$$\nu = \begin{cases} \nu_0, & \text{for } l \leq l_0, \\ \nu[1 + \varepsilon(l - l_0)^2], & \text{for } l > l_0. \end{cases}$$

- we choose  $l_0 = 85, \varepsilon = 10^{-2}$
- The time integration is performed using the Crank-Nicolson scheme for the diffusion terms and the second-order Adams-Bashforth scheme for the other terms.

# Outline

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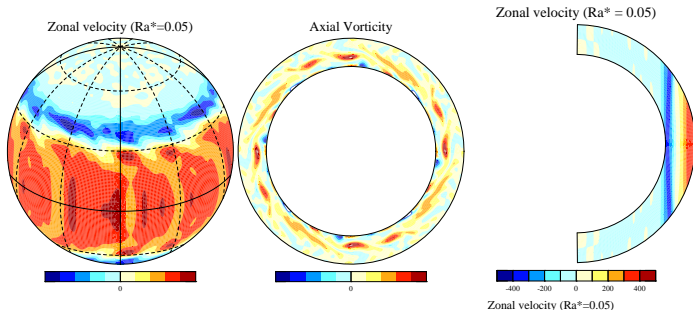
Background

Formulation and experiments setup

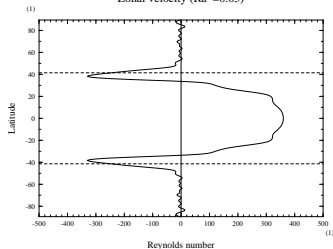
**Results**

Summary

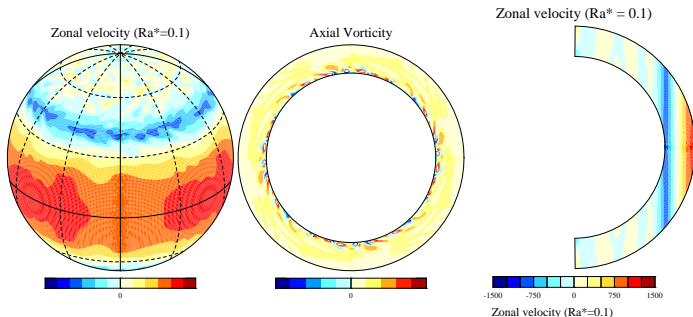
# Results: $Ra^* = 0.05$



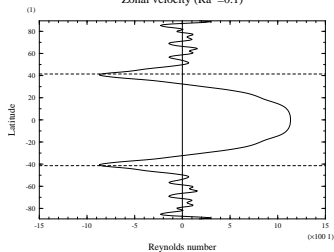
- Broad prograde equatorial jet
- Alternating zonal jets do not emerge in mid- and high-latitudes.



# Results: $Ra^* = 0.1$



- Broad prograde equatorial jet
- Mean zonal flow: alternating zonal jets emerge in mid- and high-latitudes
- Outer surface: alternating zonal jets in high latitudes ?



## Results: output parameters

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- Input parameters

parameters	present study	HA2007
Prandtl number: Pr	0.1	0.1
Radius ratio: $\eta$	0.75	0.85, 0.9
Ekman number: E	$10^{-4}$	$10^{-6}$
Modified Rayleigh number: $Ra^*$	0.05, 0.1	0.05

- Output parameters

parameters	present study	HA2007
local Reynolds number: Re	$3.59 \times 10^2, 1.13 \times 10^3$	$5 \times 10^4$
local Rossby number: Ro	$3.59 \times 10^{-2}, 1.13 \times 10^{-1}$	$1.2 \times 10^{-2}, 2.5 \times 10^{-2}$

- small Re, i.e. weak jet
- Ro, i.e. slow rotation rate

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# Summary and discussion

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## Summary

- the profile of mean zonal flow:
  - Broad prograde equatorial jet
  - Alternating zonal jets emerge in mid- and high-latitudes
- Outer surface: alternating zonal jets in high latitudes ?
  - thick shell? , small  $Ra^*$  ?, large  $E$  ?
  - hyperdiffusivity?

## In the future...

- More 'thin', 'fast rotating' spherical shell convection
  - $\eta = 0.75$   $\eta = 0.8, 0.85, 0.9$
  - $E = 10^{-4} \rightarrow E = 3 \times 10^{-5}$
- Investigation of generation mechanism
  - Comparison with the Rhines scale