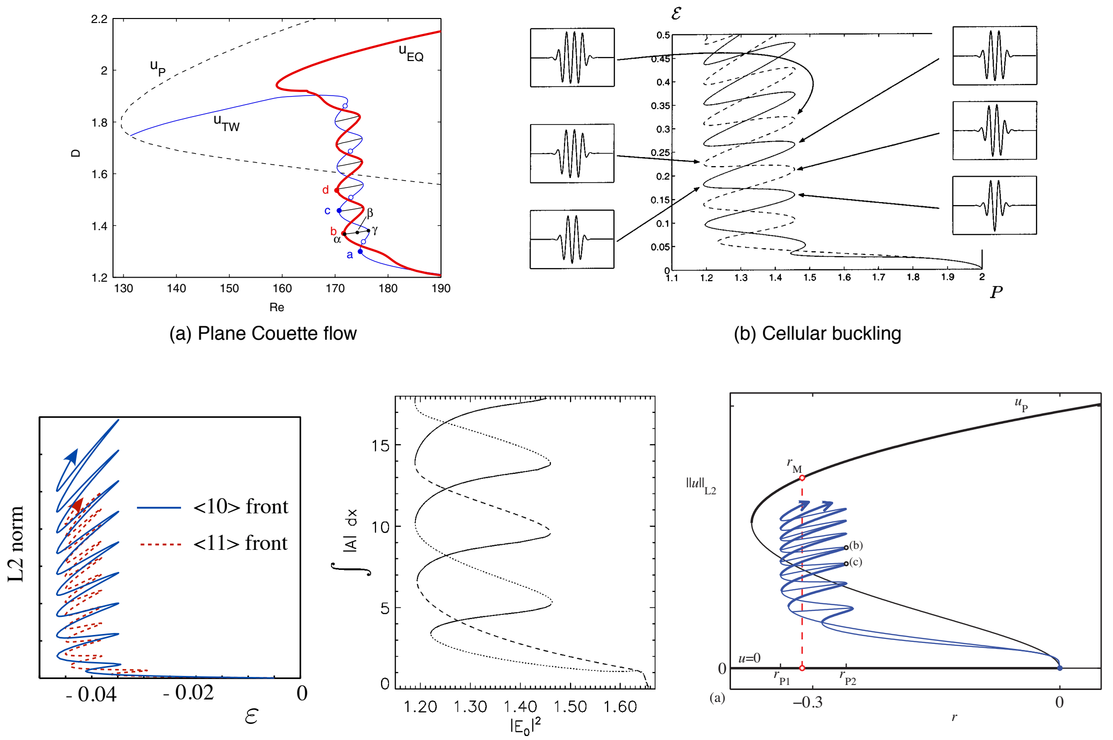


(a) Plane Couette flow (b) Cellular buckling

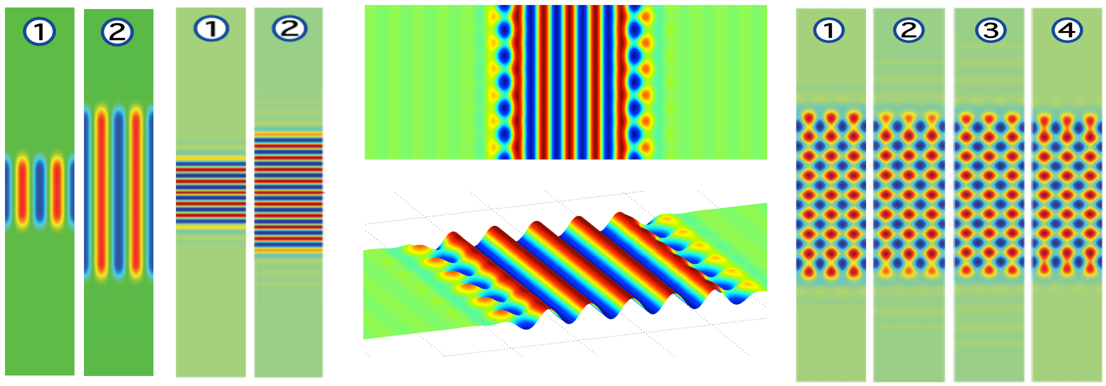
(c) Crime hotspots (d) Optical cavity solitons (e) Vegetation patches

Figure 1: Examples of bifurcation diagrams corresponding to widely disparate physical systems with similar underlying mathematical structure. Figures are reproduced from published works as follows: (a) plane Couette flow [8]; (b) cellular buckling [2]; (c) crime hotspots [3]; (d) optical cavity solitons [6]; (e) vegetation patterns [7].

Uncovering the Patterns Behind Patterns

Project description: In this project we will explore localized patterns on the plane. Such patterns appear in a wide variety of physical contexts, which include – but are not limited to! – fluid flows, crime hot spots, buckling problems, vegetation growth and optical systems. Bifurcation diagrams for widely disparate systems have proved to be remarkably similar; see Figure 1.

Various types of patterns which are periodic in one direction and localized in the other, including those termed "rolls", "spots and stripes", and "squares," have been investigated (see, for example, [1], and refer to Figure 2 for visualizations of particular patterns). We



(a) roll patterns (b) spot and stripe patterns (c) square patterns

Figure 2: Examples of patterns observed in the Swift–Hohenberg equation on an infinite cylinder. See [1] and [5].

would like to use this time and computing resources to better understand the relationships between these patterns, and, in particular, their connections in parameter space.

As time and interest allow, we may also explore the formation of large localized hexagon patches. Computing resources were specifically identified as a limiting factor in a 2008 study [4]. However, since this time the underlying package capabilities have progressed substantially, and revisiting this problem with new computing resources may enable substantial progress.

Prerequisites: While there are no formal prerequisites for participation in this project, some familiarity with differential equations and dynamical systems theory would be helpful. In particular, a basic understanding of bifurcation theory underlies most of the work. I would be happy to review with any interested students lacking this background; Strogatz [9] also provides an accessible and useful introduction. Programming for this project will be conducted in C/C++. Current software is written in Matlab, so our first task will be converting this software. Depending on our progress we may also use Auto07p, a Fortran based program for continuation, but no knowledge of Fortran is expected.

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