

Lecture 1+: Introduction, Dry Convection, Meteorological Basics and Moist Thermodynamics

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Overview

- 1 Introduction: Why do we care about moist convection?
- 2 Rayleigh-Benard Convection
- 3 Basics of the compressible atmosphere
- 4 A little basic geophysical fluid dynamics
- 5 Moist thermodynamics

Introduction, 1: Why do we care?

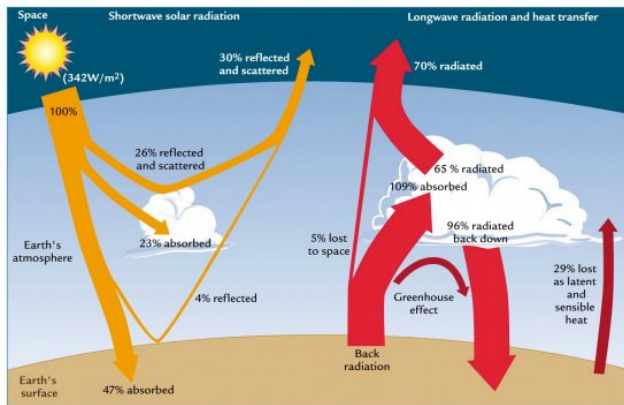
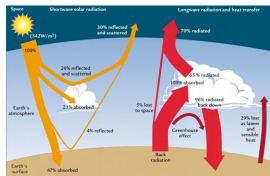


Image from Jean Lynch-Stieglitz, Georgia Tech

Introduction, 1: Why do we care?



- *Net* infrared radiative flux at earth's surface is relatively small; there is a rough balance between insolation and latent + sensible heat flux;
- Latent \gg sensible globally (and over oceans especially);
- Infrared emission to space (balancing all non-reflected sunlight at top of atmosphere) comes mostly from high altitude;
- **Moist convection is essential** for moving energy from the surface to high altitude where it can be emitted to space.

Also, moist convection controls the morphology of weather on earth.

Pictures of clouds

Adam Sobel

FDEPS Kyoto

stratocumulus



Rob Wood, U. Washington

stratocumulus



www.atmos.washington.edu/~robwood

Rob Wood, U. Washington

shallow cumulus
(non-precipitating)



shallow cumulus
(non-precipitating)



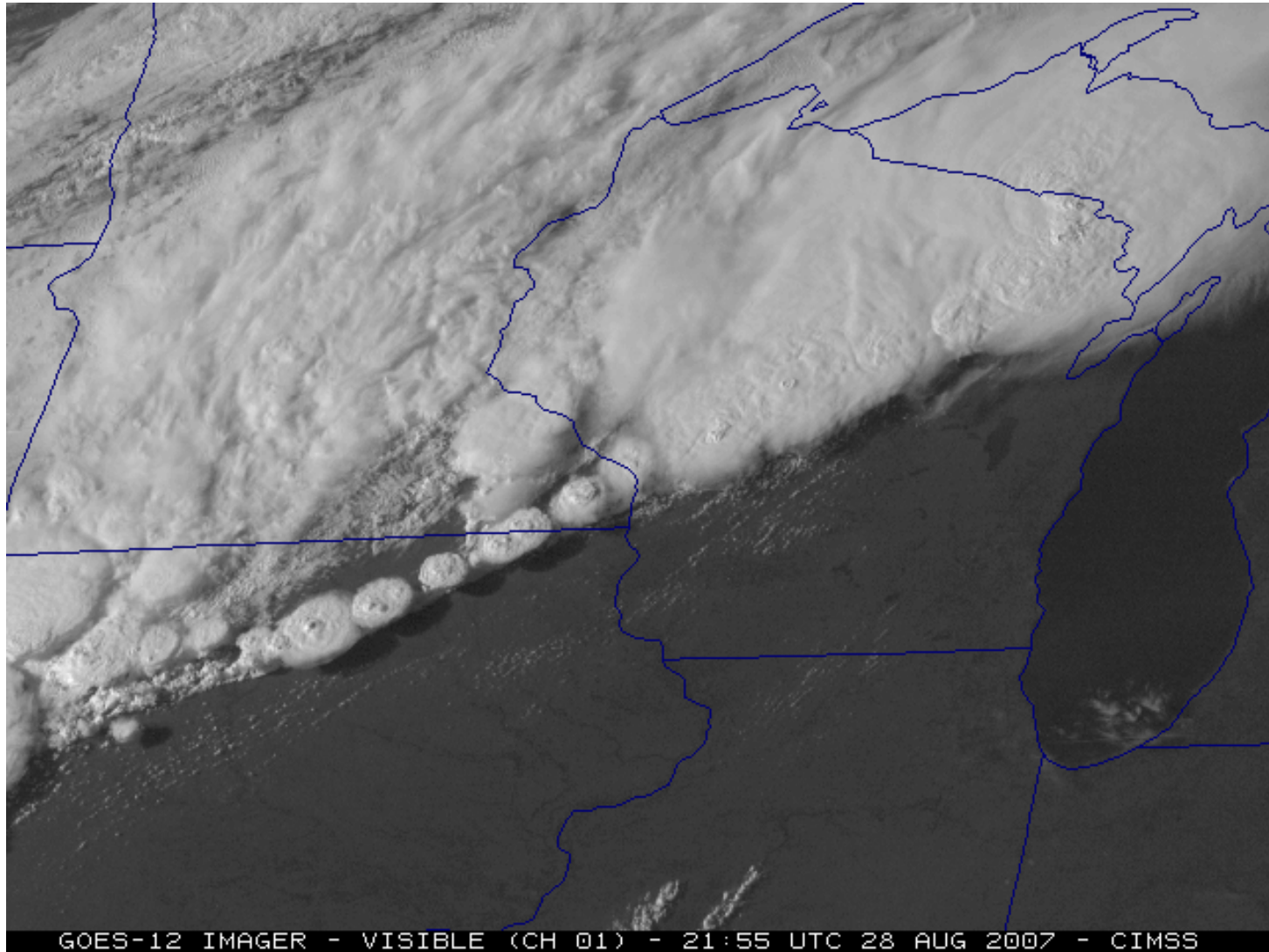
Isolated precipitating cumulus



Isolated precipitating cumulus
(more developed, note anvil)



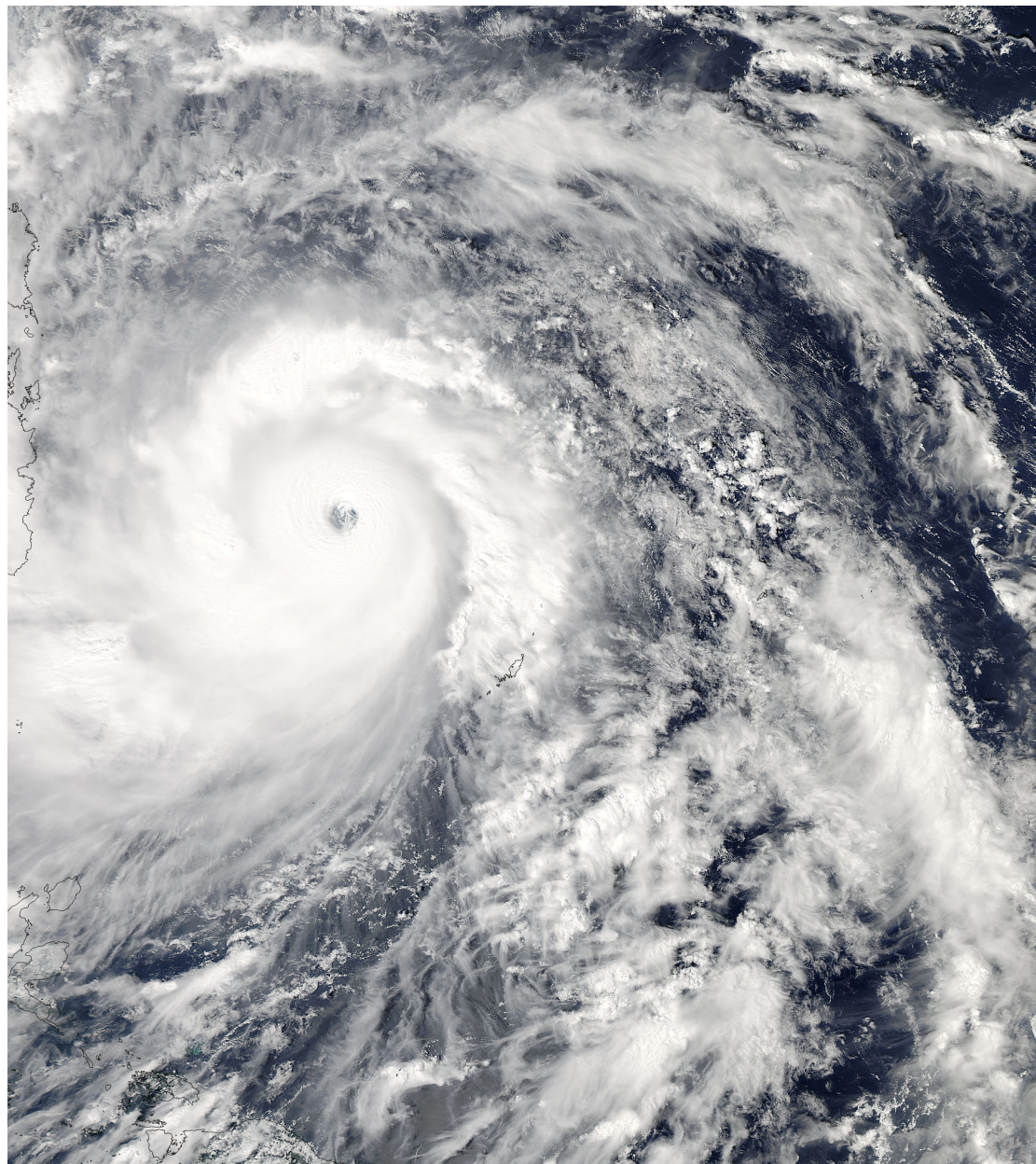
Mesoscale convective system in visible satellite image



<http://cimss.ssec.wisc.edu/goes/blog/archives/494>

Super Typhoon Haiyan

Image: NASA





John Allen, Columbia

This treatment of dry convection between parallel plates (Rayleigh-Benard) is taken from Kundu, *Fluid Mechanics*, but also draws from Emanuel, *Atmospheric Convection* (1994).

Rayleigh-Benard convection, 1

Boussinesq fluid with Cartesian velocity component \tilde{u}_i , pressure \tilde{p} , temperature \tilde{T} . Two parallel plates, infinite in horizontal extent, separated by distance d ; one is at $z = -d/2$, other at $z = +d/2$. Lower plate is at temperature $\tilde{T} = T_0$, upper at $\tilde{T} = T_0 - \Delta T$. Linear equation of state, thermal expansion coefficient α , thermal diffusivity κ , viscosity ν .

$$\rho = \rho_0[1 - \alpha(T - T_0)] \quad (1)$$

$$\frac{d\tilde{\mathbf{u}}}{dt} = -\frac{1}{\rho_0}\nabla\tilde{p} - g[1 - \alpha(\tilde{T} - T_0)]\hat{k} + \nu\nabla^2\tilde{u}_i \quad (2)$$

$$\frac{d\tilde{T}}{dt} = \kappa\nabla^2\tilde{T} \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (4)$$

Rayleigh-Benard convection, 2

Now define perturbations about a basic state of no motion as variables without tildes:

$$\tilde{u}_i = u_i(\mathbf{x}, t) \quad (5)$$

$$\tilde{T} = \bar{T}(z) + T(\mathbf{x}, t) \quad (6)$$

$$\tilde{p} = P(z) + p(\mathbf{x}, t) \quad (7)$$

The basic state must satisfy

$$0 = -\frac{1}{\rho_0} \nabla P - g[1 - \alpha(\bar{T} - T_0)]\hat{k}$$

with \hat{k} the vertical unit vector. Thus

$$\frac{1}{\rho_0} \frac{\partial P}{\partial z} = g[1 - \alpha(\bar{T} - T_0)]\hat{k} \quad (8)$$

$$0 = \kappa \frac{\partial^2 \bar{T}}{\partial z^2} \rightarrow \bar{T} = T_0 - \Gamma(z + d/2), \quad \Gamma = \frac{\Delta T}{d}. \quad (9)$$

Is this state stable to small perturbations?

Rayleigh-Benard convection, 3

Subtract the equations for the basic state from the total equations, and linearize, assuming small perturbations, to obtain

$$\frac{\partial u_i}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_i} + g\alpha T \hat{k} + \nu \nabla^2 \tilde{u}_i \quad (10)$$

$$\frac{\partial T}{\partial t} - \Gamma w = \kappa \nabla^2 \tilde{T} \quad (11)$$

$$\nabla \cdot \mathbf{u} = 0. \quad (12)$$

The boundary conditions are (using no-slip):

$$u = v = w = T = 0, \quad z = \pm d/2.$$

Also, by continuity $\frac{\partial w}{\partial z} = 0$ at the walls. (Or, we can use stress-free, $\frac{\partial^2 w}{\partial z^2} = 0$.)

Rayleigh-Benard convection, 4

Now in steady state the advection and diffusion terms must balance, so scaling analysis requires that

$$\Gamma w \sim \frac{\kappa T}{d^2},$$

but $T \sim \Gamma d$, so

$$w \sim \frac{\kappa}{d}.$$

There are two non-dimensional parameters. The ratio of the buoyancy to viscous forces is:

$$\frac{g\alpha T}{\nu w/d^2} \sim \frac{g\alpha \Gamma d}{\nu w/d^2} \sim \frac{g\alpha \Gamma d^4}{\nu \kappa} \equiv Ra,$$

the *Rayleigh number*. Then the other is the ratio of the diffusivities, or *Prandtl number*

$$Pr = \frac{\nu}{\kappa}.$$

Now we **skip a lot of math**, summarizing the steps in words. By substitution we can reduce the equations to just two, for w and T . Use the method of normal modes. Since the PDEs have constant coefficients, we can assume exponential solutions:

$$w = \hat{w}(z) \exp(ikx + ily + \sigma t), \quad (13)$$

$$T = \hat{T}(z) \exp(ikx + ily + \sigma t), \quad (14)$$

where σ is complex, $\sigma = \sigma_r + i\sigma_i$. \hat{T} and \hat{w} are also complex. It is useful to define $K = \sqrt{k^2 + l^2}$. (Note there is rotational symmetry, x and y are interchangeable.)

The first key result is that σ is **pure real** in this problem, $\sigma_i = 0$. There are no oscillatory solutions. This means that the **marginal stability curve** (in K, Ra space) is that for which $\sigma = 0$ (rather than just $\sigma_r = 0$).

Rayleigh-Benard convection, 6

We do not solve the full stability problem. Rather, we find the value of K for which $\sigma = 0$ at a given Ra .

- Problem reduces to a 6th order linear elliptic PDE with 6 homogeneous BCs.
- We non-dimensionalize distance by d , T by ΔT , w by $\frac{\kappa}{d}$.
- Pr drops out of the problem (for marginal stability).
- Because the BCs are symmetric, the eigenfunctions in z can be either symmetric or antisymmetric about the midpoint $z = 0$.
- The smallest Ra for which $\sigma = 0$ — the value above which we expect to find motion — comes from the gravest even mode. This is verified by experiment.
- The problem is mathematically much simpler if we use **stress-free** BCs; the results are quantitatively, but not qualitatively different from the no-slip case (in all ways other than the horizontal velocity at the boundary) so we focus on that case.

Rayleigh-Benard convection, 7

The even solution for the stress-free eigenfunctions is

$$w = A \sin n\pi z, \quad n = 1, 2, 3 \dots$$

The critical Rayleigh number is

$$Ra = \frac{(n^2\pi^2 + K^2)^3}{K^2}.$$

The lowest value this can take on for any K is found by differentiating with respect to K ; the result is

$$K_{cr} = \frac{\pi^2}{2}.$$

This is (almost) $O(1)$; since we have non-dimensionalized by d , **the horizontal scale of the motion is the same order as the vertical scale.**

The Rayleigh number at which marginal stability of the system occurs is

$$Ra_{cr} = \frac{27\pi^4}{4} \approx 657,$$

(somewhat larger for no-slip, but same order of magnitude). For $Ra < Ra_{cr}$, there will be no motion; heat will be transported diffusively from bottom to top.

The horizontal structure of the circulation cells at marginal stability is not predicted by the theory, since only K , not k or l separately, appears. Any regular polygons are possible. The detailed pattern is somewhat sensitive to experimental details, but often begins as hexagons, and forms long rolls later.

youtube video

Rayleigh-Benard convection, 9

Taken from Emanuel, 1994. (pp.84)

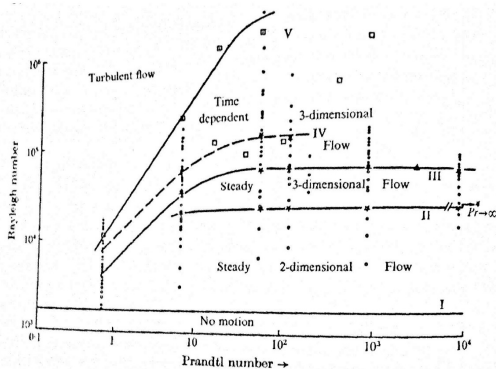


Fig. 3.12 Regime diagram for experiments on Rayleigh convection. Circles represent steady flows and circular dots denote time-dependent convection. The stars represent transition points. The open squares show independent laboratory observations of time-dependent flow by Rossby (1966) and the squares with a dot in the center show observations of turbulent flow by Willis and Deardorff (1967). [From Krishnamurti (1970b).]

Rayleigh-Benard convection, 10

In the atmosphere, the Rayleigh numbers are *MUCH* higher than critical — by many, many orders of magnitude. (Diffusivities are small, length scales are large, heat flux also large.) So it is of interest to say something about large Ra .

The flow quickly becomes turbulent as one gets much past critical.

youtube video of turbulent 2D convection

The *Nusselt number*, or nondimensional heat flux,

$$Nu = \frac{\overline{wT}}{\kappa\Delta T}$$

increases with Ra , in a somewhat complex way (several regimes of different turbulent flow types).

The details of all the turbulent transitions in Nu are not really relevant to the atmosphere, because the heat flux in the atmosphere is more nearly constrained externally than is the temperature.

For our purpose the main quantity of interest in the turbulent solutions is the mean temperature, $\overline{T}(z)$. This is close to homogeneous in the interior, with all the gradient trapped in very thin layers near the boundaries.

Stable stratification between parallel plates,1

Now consider same situation, but with the top boundary warmer than the bottom. Then it is completely stable. Just for sake of argument consider the linear equations in 2D with both diffusivities set to zero:

$$\begin{aligned}\frac{\partial u}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial x}, \\ \frac{\partial w}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial z} + \alpha g T, \\ &\frac{\partial T}{\partial t} + w \Gamma, \\ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0\end{aligned}$$

where now $\Gamma = (T(d/2) - T(-d/2))/d > 0$.

Stable stratification between parallel plates, 2

The eigenfunctions are the same, and we can show that we get pure neutral oscillatory gravity wave solutions,

$$w, T \sim \sin(n\pi z) \exp(i(\omega t + kx)),$$

with frequency

$$\omega^2 = \frac{N^2 k^2}{k^2 + n^2 \pi^2},$$

where

$$N^2 = \alpha g \Gamma$$

defines the **buoyancy frequency**, N .

Stable stratification between parallel plates, 3

Since for the disturbances of interest in the real atmosphere have horizontal scales much longer than vertical scales, the most relevant analogs in the parallel-plate system are also those with $k \ll n$, so

$$\begin{aligned}\omega^2 &= \frac{N^2 k^2}{k^2 + n^2} \approx \frac{N^2 k^2}{k^2 + n^2 \pi^2}, \\ \omega &\approx \frac{Nk}{n\pi}.\end{aligned}\tag{15}$$

Thus modes of higher vertical wave numbers (shorter vertical scales) propagate more slowly. This approximation is equivalent to assuming hydrostatic balance.

Stable stratification between parallel plates, 4

The phase speed of wave modes $c = \omega/k$, so

$$c = \frac{N}{n\pi},$$

but remember that we have nondimensionalized distance by d .
Dimensionally,

$$c = \frac{Nd}{n\pi}.$$

To fix numbers, for the atmosphere $N \approx 2 \times 10^{-2} \text{s}^{-1}$, and we may take the vertical scale d to be the depth of the troposphere, 10 km. That is broadly consistent with the value $\mathbf{c} \approx \mathbf{50m/s}$ for waves with $n = 1$.

A few points about internal gravity waves

- 1 If there were no boundaries, they would also propagate vertically. (The atmosphere has no upper lid, though for some purposes can behave as though it almost does.) The vertical wave numbers would not be quantized in that case.
- 2 Putting the diffusivities back in would just make them weakly damped.
- 3 Horizontal structure: w and T are in quadrature — as with convective solutions, but opposite phasing (upward motion cooling vs. warming).

Basics of the compressible atmosphere, 1

Ideal gas law:

$$p = \rho RT,$$

with p pressure, ρ density, T temperature, R gas constant. On large scales it is a very good approximation to assume hydrostatic balance in the vertical:

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{pg}{RT}.$$

T is measured in Kelvin and doesn't vary that much compared to its absolute value, so pressure drops approximately exponentially with height:

$$p \sim p_0 e^{z/H}$$

where $H = RT_0/g$ is the **scale height**, T_0 a reference temperature.

We can also show that the **potential temperature**,

$$\theta = T \left(\frac{p_0}{p} \right)^\kappa,$$

with $\kappa = R/c_p = 2/7$, is conserved in the absence of friction or heating of the dry fluid. It is related to specific entropy by $s = c_p \ln \theta + \text{constant}$.

Basics of the compressible atmosphere, 2

Because of hydrostatic balance on large horizontal scales, we can use pressure as a coordinate. In this coordinate the vertical velocity is

$$\omega = \frac{dp}{dt},$$

and the mass conservation equation is isomorphic to the incompressible one (although we do not assume incompressibility)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = \nabla \cdot \mathbf{u} = 0.$$

In the horizontal momentum equation, instead of the pressure gradient on a constant height surface, we have to write the gradient of geopotential height, $\phi = gz$, on a constant pressure (isobaric) surface. The geopotential height is found by integrating hydrostatic balance using the actual temperature

$$\phi = \int g dz = \int_p^{p_s} \frac{RT}{p} dp = \int_p^{p_s} RT d \ln p.$$

Basics of the compressible atmosphere, 3

Including Coriolis parameter $\mathbf{f} = (2\Omega \sin \lambda)\hat{z}$, ($\lambda =$ latitude) the **primitive equations** in p coordinates are:

$$\frac{d\mathbf{v}}{dt} + \mathbf{f} \times \mathbf{v} = -\frac{1}{\rho} \nabla_p \phi + \mathbf{F}$$

$$\frac{\partial \phi}{\partial p} = \alpha = -\frac{1}{\rho}$$

$$\frac{d\theta}{dt} = Q$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

$$\frac{dq}{dt} = e - c.$$

where $\mathbf{v} = (u, v)$, $d/dt = \partial/\partial t + u\partial/\partial x + v\partial/\partial y + \omega\partial/\partial p$, and $\nabla_p \phi = \frac{\partial \phi}{\partial x} \hat{x} + \frac{\partial \phi}{\partial y} \hat{y}$, and all horizontal derivatives are understood to be taken along isobaric surfaces. Heating is Q , friction is \mathbf{F} , q specific humidity (defined soon), evaporation and condensation of water are e, c .

Basics of the compressible atmosphere, 4

We are working in a local Cartesian planar approximation, so there are no metric terms, and horizontal coordinates are x, y instead of latitude and longitude.

We will represent the Coriolis parameter by the β - plane approximation (first two terms in Taylor expansion of the sine function),

$$f = f_0 + \beta y,$$

where $f = 2\Omega \sin \lambda_0$ and $\beta = \frac{df}{dy} = (2\Omega \cos \lambda_0)/a$, a is earth radius, λ_0 a reference latitude.

If we wish, instead of using potential temperature, we can use **dry static energy**,

$$s = c_p T + gz$$

(not to be confused with entropy although the letter s is also used). s is conserved for dry adiabatic transformations in which hydrostatic balance is maintained, and thus for large-scale motions we can write

$$\frac{ds}{dt} = Q$$

Basics of the compressible atmosphere, 5

The primitive equations describe the motions on large scales. On sufficiently small horizontal scales ($L \leq 10$ km) hydrostatic balance may break down. We may consider the vertical momentum equation for **convective motions** to be (in height coordinates)

$$\frac{dw}{dt} = -\frac{\partial p}{\partial z} - \rho g,$$

or defining a hydrostatically balanced component

$$\frac{\partial \bar{p}}{\partial z} - \bar{\rho} g,$$

then

$$\frac{dw}{dt} = -\frac{\partial p'}{\partial z} - \rho' g,$$

where $p = \bar{p} + p'$, $\rho = \bar{\rho} + \rho'$.

In **parcel theory**, we will neglect the pressure term, and consider just

$$\frac{dw}{dt} = -\rho' g.$$

Basics of the compressible atmosphere, 6

While temperature decreases with height in the atmosphere, potential temperature increases with height. This means the atmosphere is **stable** to small vertical displacements — at least as long as condensation of water vapor does not occur.

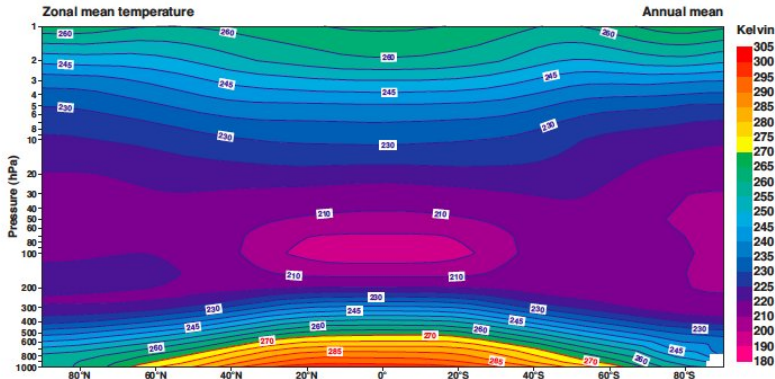
The buoyancy frequency in the compressible atmosphere, N , is

$$N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z},$$

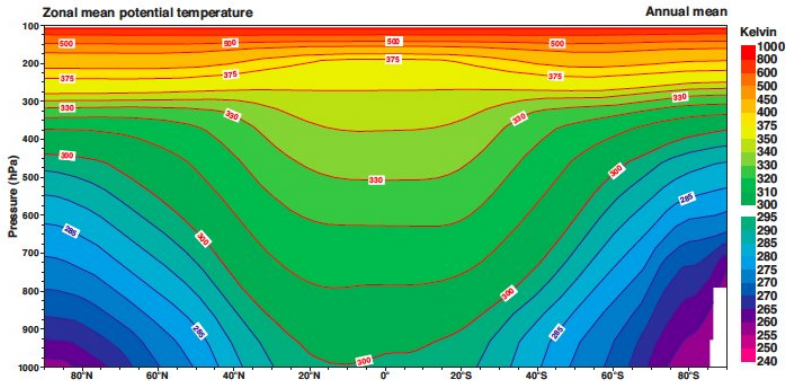
or in pressure coordinates,

$$N^2 = -\frac{\rho g^2}{\theta} \frac{\partial \theta}{\partial p}.$$

Annual mean temperature vs. latitude and height (source: ECMWF, ERA40 Rreanalysis).



Annual mean potential temperature vs. latitude and height (source: ECMWF, ERA40 Rreanalysis).



A little basic GFD, 1

If we neglect friction, time rate of change, and inertia, the horizontal momentum equation becomes

$$\mathbf{f} \times \mathbf{v} = -\frac{1}{\rho} \nabla_p \phi,$$

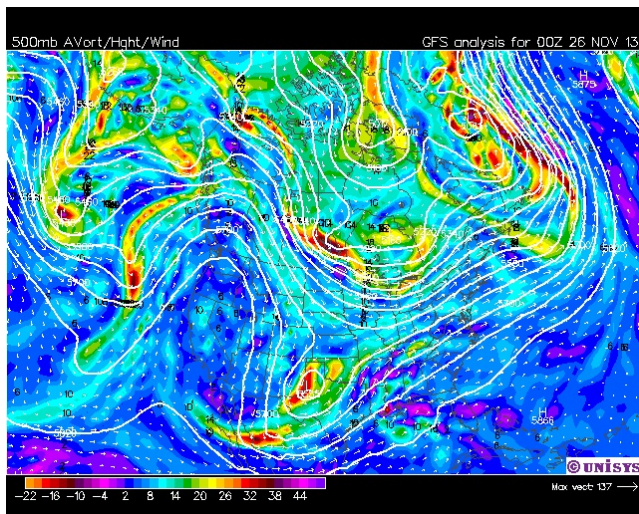
or

$$\begin{aligned}fu &= -\frac{\partial \phi}{\partial y} \\-fv &= -\frac{\partial \phi}{\partial x}\end{aligned}$$

which is **geostrophic balance** — flow parallel to isobars.

A little basic GFD, 2

500 hPa geopotential height, wind and vertical vorticity ($\partial v/\partial x - \partial u/\partial y$) over USA one day last week



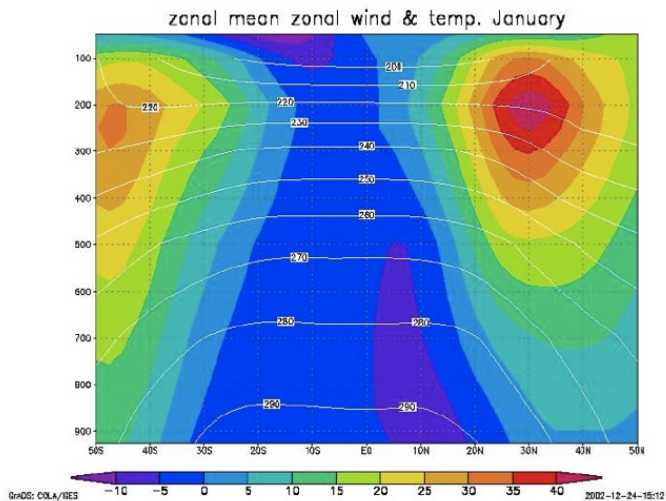
Combining geostrophic and hydrostatic balance with the ideal gas law leads to the **thermal wind relation**

$$\frac{\partial u}{\partial p} = \frac{R}{f p} \frac{\partial T}{\partial y},$$
$$\frac{\partial v}{\partial p} = -\frac{R}{f p} \frac{\partial T}{\partial x},$$

which says that the **vertical shear of the horizontal wind is proportional (and perpendicular) to the horizontal temperature gradient.**

Note the factor $\sim 1/f$ on the RHS, which implies that to the extent thermal wind balance holds, **large horizontal temperature gradients cannot exist in the tropics**, unless accompanied by very large shears.

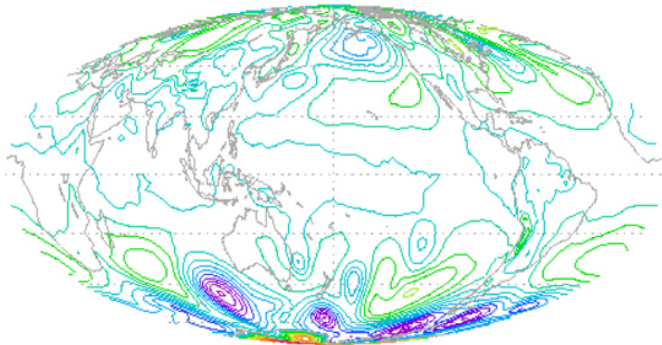
A little basic GFD, 4



A little basic GFD, 5

Surface pressure map (contour interval = 5 hPa) from a random day, May 15, 2005.

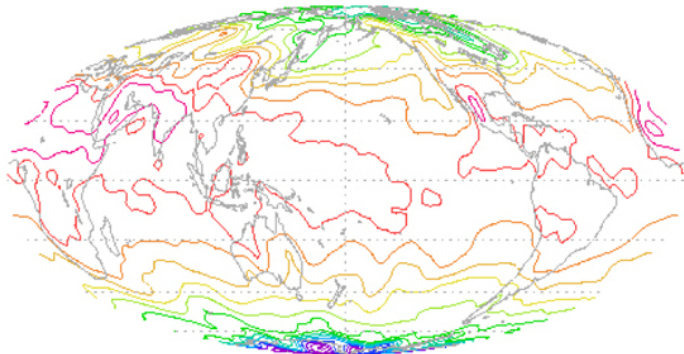
Mean sea level pressure



A little basic GFD, 6

Surface temperature map (cint=5C) from a random day, May 15, 2005.

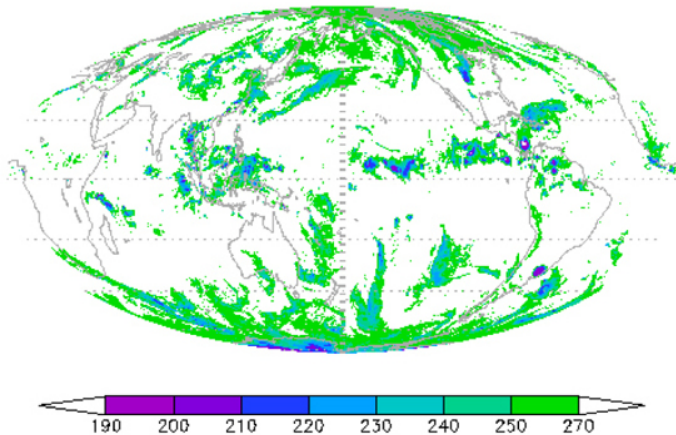
Surface temperature



A little basic GFD, 7

Effective black body temperature (C) seen from space, from a random day, May 15, 2005.

Black body temperature



A little basic GFD, 8

The natural horizontal scale for motions which are close to geostrophic balance is the **deformation radius**, or Rossby radius, L_d ,

$$L_d = \frac{c}{f}$$

where c is the gravity wave speed. (Thus motions with different vertical scales have different L_d . This scale emerges, for example, from the **geostrophic adjustment problem**.)

For tropical dynamics we may take the reference latitude in our β -plane approximation to be $\lambda_0 = 0$, thus $f = \beta y$ is the equatorial β -plane approximation. In this case the natural scale that emerges is the **equatorial deformation radius**,

$$L_\beta = \sqrt{c/\beta}.$$

This treatment of moist thermodynamics draws on Emanuel's *Atmospheric Convection* as well as Bohren and Albrecht's *Atmospheric Thermodynamics*.

basic moisture variables, 1

Basic variables to describe the water vapor content of the atmosphere include

- **mixing ratio** r , which is the **mass of water vapor divided by mass of dry air**;
- **specific humidity** q , which is the mass of water vapor **divided by the total mass of air (dry air plus water vapor)**;
- **vapor pressure** e , which is the **partial pressure of water vapor**;
- **vapor density** ρ_v , which is the mass of water vapor per unit volume.
- Then, $r = \rho_v / \rho_d$, where ρ_d is dry air density;
 $q = \rho_v / (\rho_d + \rho_v) = r / (1 + r)$.

In the earth's atmosphere, since it is always the case that $r \ll 1$ (maximum values are a few percent), $q \approx r$.

Analogously to r , we can also define the atmospheric content of condensate — liquid and ice. These are assumed to be in the form of small particles falling at their terminal velocities. If the terminal velocities are small compared to the vertical air velocity, we call them **cloud water** or **cloud ice**; if the fall velocities are comparable to or greater than the air velocity we call them **precipitation** (rain, snow, graupel, hail).

- **liquid water mixing ratio** r_l ,
- **ice mixing ratio** r_i ,
- **total water**, $r_T = r + r_l + r_i$.

We can define also specific humidities of liquid and ice analogously to vapor.

basic moisture variables, 3

Some more relationships between variables will be useful. Using the ideal gas law separately for both dry air and vapor, with gas constants R_v and R_d we can write

$$r = \frac{e/R_v T}{p_d/R_d T} = \frac{R_d}{R_v} \frac{e}{p - e} = \epsilon \frac{e}{p - e} \approx \epsilon \frac{e}{p}$$

(since we can assume $e \ll p$). Here p is total pressure, p_d dry air pressure, and

$$\epsilon = \frac{R_d}{R_v} \approx 0.622.$$

We also define the **relative humidity** \mathcal{H} :

$$\mathcal{H} = \frac{e}{e^*}$$

Where e^* is the **saturation vapor pressure**. The saturation vapor pressure is the maximum vapor pressure which can exist in equilibrium over a liquid water surface (or a solid ice surface).

The saturation vapor pressure e^* is a function of temperature, defined by the (nearly exact) relation:

$$\frac{de^*}{dT} = \frac{L_v e^*}{R_v T^2}, \quad (16)$$

where L_v is the **latent heat of vaporization**. (16) is the **Clausius-Clapeyron relation**. L_v is itself a function of temperature, but a weak one; in many cases we can assume it constant. An approximate integration yields

$$e^* = 6.112 \exp\left(\frac{17.67 T}{T + 243.5}\right),$$

where T is in Celsius (not Kelvin) and e^* is in hPa. This is valid over liquid; a similar relation holds over ice, but with different numbers.

It is also useful to define the **dew point**, or temperature to which a given parcel would have to be cooled at constant pressure in order for saturation to occur. Dew point is a function of specific humidity and pressure.

The difference between the temperature and dew point, $T - T_d$, is sometimes called the dew point depression. It is related to **saturation deficit**, which may be defined $e^* - e$, $q^* - q$, or $r^* - r$.

phase changes of water in the atmosphere,1

When $e > e^*$, condensation is possible. (Any excess $e - e^* > 0$ is called **supersaturation**.) Water vapor can condense into liquid or solid particles by **homogeneous nucleation** (spontaneous clumping of water molecules), or **heterogeneous nucleation** (condensation onto a pre-existing liquid or solid particle). Pre-existing particles on which condensation can occur are called **cloud condensation nuclei** (CCN). Heterogeneous nucleation will occur first if CCN are present.

In the atmosphere, there are always some CCN present for condensation into liquid. Homogeneous nucleation of liquid droplets is believed not to occur, nor is significant supersaturation over liquid. One can assume $e \leq e^*$.

phase changes of water in the atmosphere, 2

On the other hand, **ice nuclei**, onto which either liquid drops or water vapor can freeze, are not so universally present. Significant **supercooling** (existence of liquid water at $T < 0^\circ\text{C}$) can occur, as can significant supersaturation of vapor with respect to ice.

At temperatures $T < -40^\circ\text{C}$, homogeneous freezing of liquid takes place and all condensed water can be assumed to be ice. However in the range $-40^\circ\text{C} < T < 0^\circ\text{C}$, the phase of condensate may not be known *a priori*, and depends on the presence or absence of ice nuclei.

At $T > 0^\circ\text{C}$, melting of ice takes place quickly. (This can be visualized in radar images by the thin **bright band**, we will see later.)

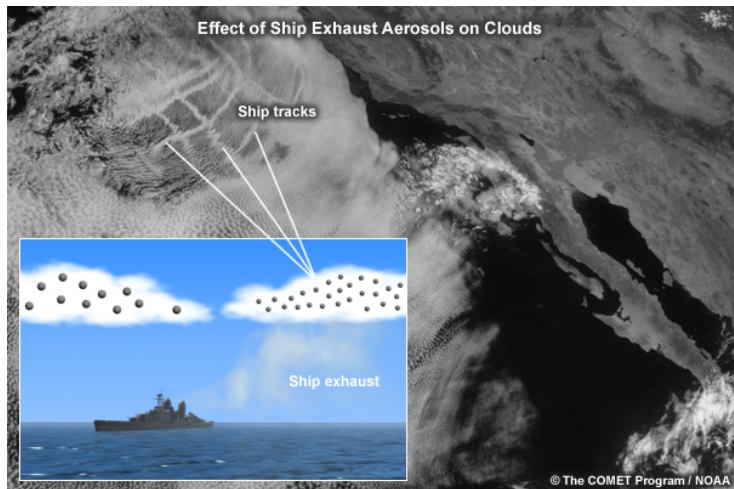
phase changes of water in the atmosphere, 3

Although there are always enough CCN for homogeneous nucleation, variations in the distribution of CCN can be important to the properties of any clouds that form.

Where there are few **aerosols** (atmospheric liquid or solid particles), cloud drops will be fewer and larger for the same amount of liquid. Where the number density of aerosols is higher, cloud drops will be greater in number and smaller in size.

In aerosol-rich environments, clouds may be **optically thicker** — more reflective of sunlight — and less likely to form precipitation (large drops). These effects are most apparent in shallow, non- or weakly-precipitating clouds.

phase changes of water in the atmosphere, 4



Effect of water on air density, 1

Water vapor has a different heat capacity than does dry air, and thus the mixture of the two also has a different heat capacity than dry air would (slightly, since $e \ll p$, $\rho_v \ll \rho_d$ etc.). The effective gas constant for the mixture is

$$R' \equiv R_d \frac{1 + r/\epsilon}{1 + r} > R_d$$

The margin by which R' exceeds R_d depends on the humidity. The mixture obeys the ideal gas law

$$p = \rho R' T$$
$$\alpha = \frac{R' T}{p},$$

where $\alpha = 1/\rho$ is the specific volume.

Effect of water on air density, 2

The direct effect of water vapor on air density is usually measured by the effective temperature which dry air would need to have in order to have the same density as moist air with a particular temperature and humidity. This is called the **virtual temperature**, T_v , defined by

$$R_d T_v = R' T,$$

thus

$$T_v = T \frac{1 + r/\epsilon}{1 + r} \approx T(1 + 0.61r),$$

with T now in Kelvin. The difference term $0.61rT$ is sometimes known as the "virtual temperature effect". It is small, generally, since r is.

Note that the virtual temperature effect is only the direct effect of vapor on density. **It has nothing to do with phase change.**

Effect of water on air density, 3

The **moist enthalpy** (really specific moist enthalpy)

$$k = (c_p + r_T c_l)T + L_v r \approx c_p T + L_v q$$

is conserved under phase changes between liquid and vapor at constant pressure. (c_l is liquid water heat capacity).

Thus the temperature increase for condensing a quantity Δq of vapor is

$$\Delta T = L_v \Delta q / c_p.$$

We can now show that compared to the virtual temperature effect, **phase change is by far the more important process by which water vapor influences the atmospheric circulation.**

Effect of water on air density, 4

The virtual temperature effect is $0.61rT$; on earth $r \leq$ a few percent. Let $r = 0.03$, then for $T = 300K$, the virtual temperature effect is $\sim 5K$. This seems large, but water vapor gradients tend not to be large, and in any case the dynamics will simply adjust; the pressure field simply hydrostatically adjusts to T_v instead of T .

By contrast condensation tends to be localized, and much larger. The effect on temperature of condensing all the water in our parcel with $r = 0.03$ is

$$L_v r / c_p \approx 75K!$$

It will not all condense at once, but the enthalpy is there.

Once there is condensate in the air, we assume it to be in suspension, falling at terminal velocity. The mass of the condensate then simply can be considered to contribute directly to the density of the air. This effect is called **condensate loading**.

Condensate loading, particularly by large precipitation particles, can be important in the generation of **convective downdrafts**. However in most other circumstances we will ignore it.

Conserved variables under phase change, 1

Condensation and evaporation of water, if done adiabatically — slowly, in equilibrium, with the vapor always at saturation — is a thermodynamically reversible process. Thus we can define a moist entropy which is conserved. That entropy can be related to a potential temperature - like quantity, called *equivalent potential temperature*, θ_e :

$$(c_{pd} + r_T c_l) \ln \theta_e = s + R_d \ln p_0,$$

where c_{pd} , c_l are dry air and liquid water heat capacities and p_0 is a constant reference pressure; s is the moist entropy (an expression for which we will not explicitly write here). This is the **reversible** θ_e , conserved if all water remains in the parcel. It is changed by any irreversible process — heating, friction, non-equilibrium water vapor transport (diffusion), phase changes which don't occur at equilibrium. If the air is purely dry, $r_T = 0$, it reduces to the dry potential temperature, θ .

Conserved variables under phase change, 2

The reversible θ_e is not conserved if precipitation occurs, since then water mass is irreversibly removed from the air. We define **pseudo-adiabatic** process as one in which all condensate is removed from the air as soon as it is formed. Then the pseudoadiabatic potential temperature, θ_{ep} , is the temperature reached by pseudoadiabatic ascent to $p = 0$ (so all water is removed) and then dry adiabatic descent to the reference pressure.

If atmospheric processes were fully reversible, there would be no rain (or snow). If they were fully pseudoadiabatic, there would be no clouds.

Clearly, the real atmosphere lies between these two extremes.

θ_{ep} can be computed by an approximate formula which is sufficiently complicated as to not be worth writing down. (E.g., see Emanuel, p. 132)

Conserved variables under phase change, 3

The **moist static energy** is defined as

$$h = (c_{pd} + r_T c_l) T + L_v r + (1 + r_T) g z,$$

where g is gravitational acceleration and z is altitude. h is conserved for adiabatic transformations in which mass is conserved, and in which the pressure remains hydrostatic. Since r_T can be taken small, we can (and will) usually make the approximation

$$h \approx c_{pd} T + L_v q + g z.$$

We have also defined the **dry static energy**, $s = c_{pd} T + g z$ (not to be confused with moist entropy), such that $h = s + L_v q$.

In many situations one has a choice whether to work with h or θ_e (or θ_{ep}). This is to some extent a matter of taste. h is an energy, θ_e is (basically) an entropy. In practice they are conserved (or not conserved) to comparable extents. h is algebraically simpler to use.

Because of the nonlinearity of the Clausius-Clapeyron relationship, things can happen in a moist fluid that are impossible for a dry fluid.

- ① A mixture of two unsaturated parcels can become saturated. (This is how you can see your breath on a cold day.)
- ② A mixture of a positively buoyant (but saturated) parcel and a neutral (unsaturated) parcel can become negatively buoyant. (This is one way convective **downdrafts** can form, though not the most important.)
- ③ Condensate can fall through the air (precipitation). Precipitation can generate downdrafts by
 - ① Direct momentum transfer (condensate loading);
 - ② Evaporative cooling in unsaturated air.

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