7. PARTICLE-DRIVEN FLOWS

7.1

Heavy particles

- crystals in magma chambers
- ash particles in pyroclastic flows
- silt in rivers
- sand in turbidity currents

Light particles

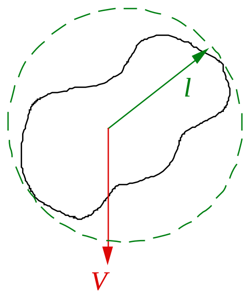
- bubbles in magma chambers
- bubbles in upper layer of oceans

Main thread:

- particle motion at low *Re*
- fluid convection at high *Re*

In low Reynolds number settling

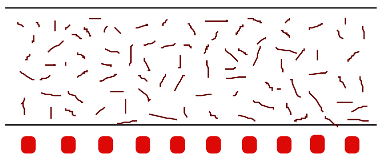
$$V = \frac{2}{9}\frac{\Delta\rho g l^2}{\mu}$$



7.2

(Koyaguchi, Hallworth, H^2 and Sparks, 1990

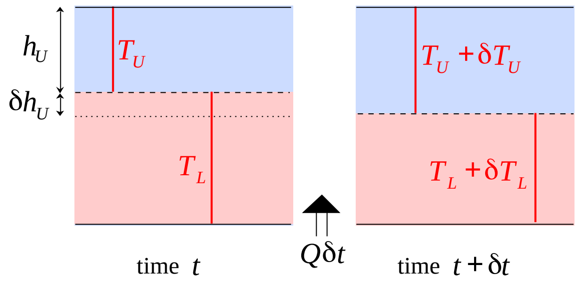
Koyaguchi, Hallworth and H^2 , 1993)



Initially uniform suspension layer of small heavy particles heated from below.

Particles sediment in a layer of increasing temperature.

7.3



$$(H - vt)\dot{T}_L = \frac{Q}{\rho c} = \beta (T_B - T_L)^{4/3} \qquad (t\dot{T}_U) = T_L$$

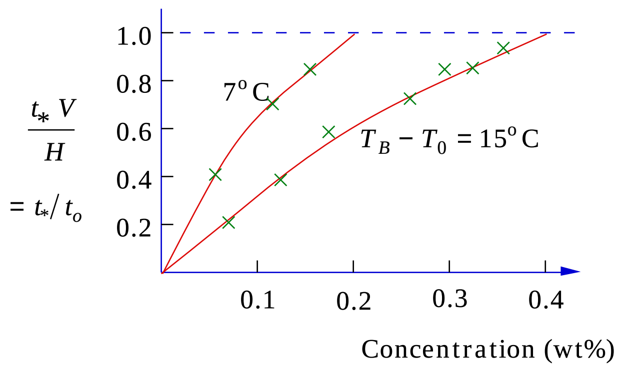
$$T_U = T_L = T_0 \quad (t = 0) \qquad \rho_L = \phi\rho_p + (1 - \phi)\rho_f$$

ρ_f, and hence ρ_L, decreases with time until $\rho_L = \rho_U$ and then overturn.

non-dimensional parameter

$$K = \Delta\rho_p/\rho_0\, \alpha\, \Delta T = \frac{\text{stabilizing influence of particles}}{\text{destabilizing influence of temperature}}$$

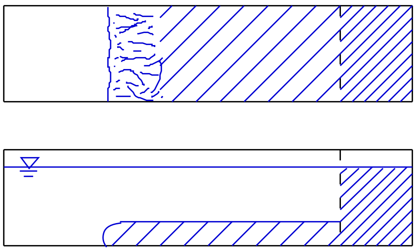
7.4



t_* : time taken to overturn

$t_o = \frac{H}{V}$: time to sink to base without convection

MOVIE OF LABORATORY GRAVITY CURRENTS



PLAN

ELEVATION

SIX SEQUENCES

A. Homogeneous Currents

Dense current

Light current

Density-layered environment

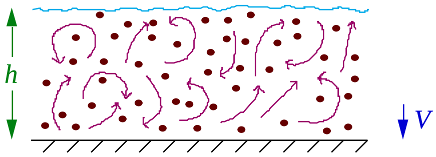
B. Particle-driven Currents

Dense current : two sequences

Light current

Light interstitial fluid

7.5



$$\delta N = -V C(0)\,\delta t \qquad (1)$$

N : number of particles in fluid column per unit area

V : Stokes free-fall velocity

$C(0)$: Concentration of particles at base

But particles are nearly uniformly distributed away from lower boundary

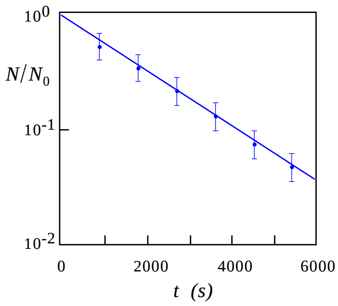
$$C(0) = N/h \qquad (2)$$

$$\dot{N} = -VN/h \qquad (3)$$

$$N = N_0 e^{-Vt/h} \qquad (4)$$

Martin and Nokes, 1988

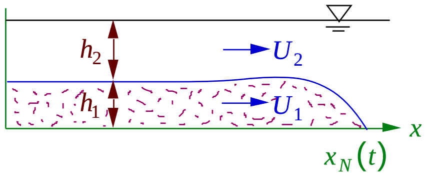
Einstein, McCave,.....



Graph of ratio N/N_0, of number of particles in suspension at time t to number of particles in suspension at time t=0, against time, t, confirming exponential decay law (4).

7.6

(Non-dimensional) Two-layer shallow water equations



Assumptions : monodisperse; vertically well-mixed; $h_1/x_N \ll 1$

$$\partial_t h_i + \partial_x(U_i h_i) = 0 \qquad (i = 1,2)$$

$$\partial_t(U_1 h_1) + h_2 \partial_x\left[U_1^2 h_1 + \tfrac{1}{2}(\phi+\gamma)h_1^2\right] - h_1 \partial_x\left(U_2^2 h_2\right) = 0$$

$$\partial_t \phi + U_1 \partial_x \phi = -\beta\phi/h_1$$

where $\gamma = \dfrac{\rho_1 - \rho_2}{(\rho_p - \rho_1)\phi_0}$ $\quad \beta = \dfrac{v_s}{\sqrt{g_0' H}}$ $\quad g_0' = \dfrac{g(\rho_p - \rho_1)\phi_0}{\rho_2}$

single particle settling velocity

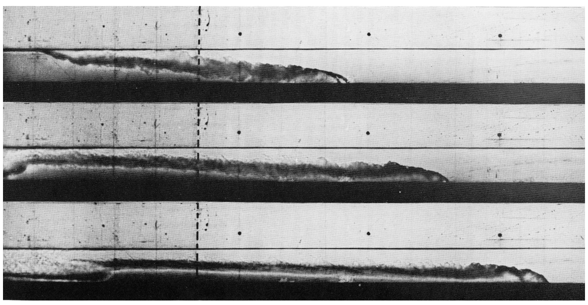
NO FREE PARAMETERS

Fixed volume : $U_1 h_1 + U_2 h_2 = 0$ Fixed flux $= U_1 h_1$

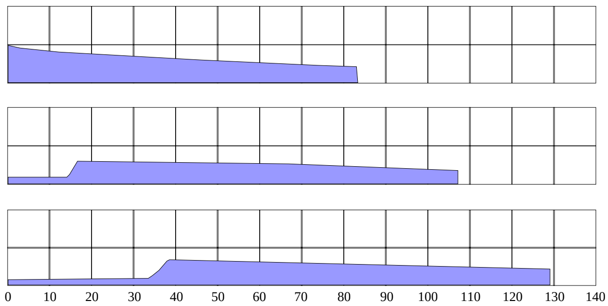
Solve numerically for h_i, ϕ, U_i fns(x,t)

(Bonnecaze, H[2] and Lister, 1993, Bonnecaze, Hallworth, H[2] and Lister, 1995)

7.7



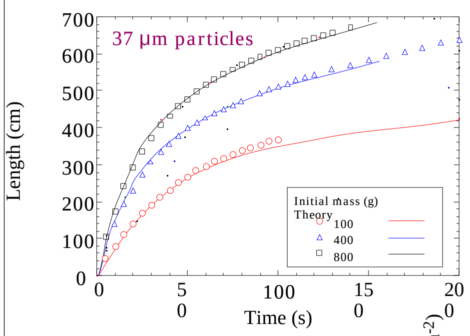
Shadowgraphs of a volume of salt water collapsing into fresh at 5, 8 and 11 seconds after release.

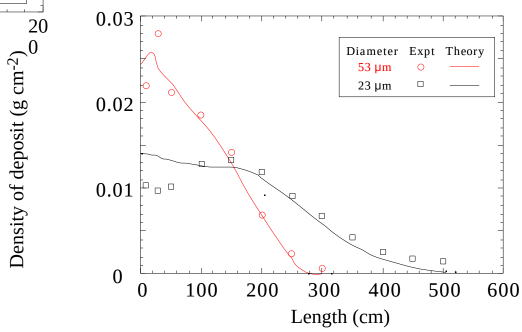


Rottman & Simpson, *JFM* **135**, 1983

Two Fluid Layers Model

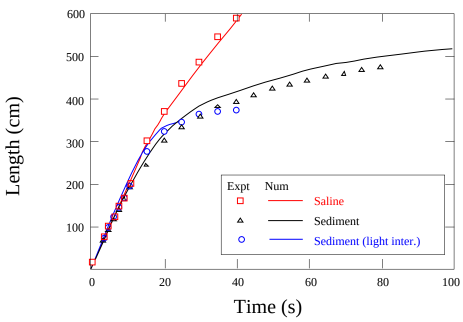
7.8

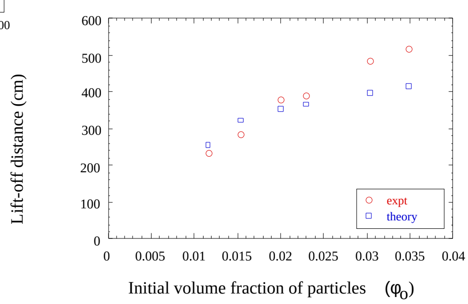




Initially dense particle-driven gravity current with buoyant interstitial fluid

7.9





Two dimensional horizontal surface

7.10

$A \equiv L^2$	initial cross-sectional area
$g'_0 \equiv LT^{-2}$	initial reduced gravity
$V_s \equiv LT^{-1}$	settling velocity of individual particle
$x_r \equiv L$	run-out length of turbidite

$$x_r = A^{1/2} f\left(A^{1/2}\, g'_o / V_s^2\right)$$

Governing equations themselves (without solution) indicate that

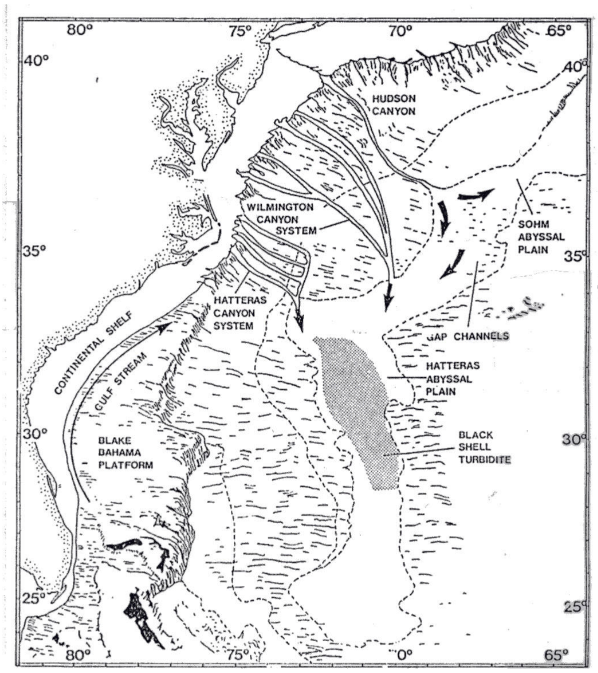
$$f(\eta) = c\, \eta^{1/5}$$

(Analytic) solution of approximate equations indicate that

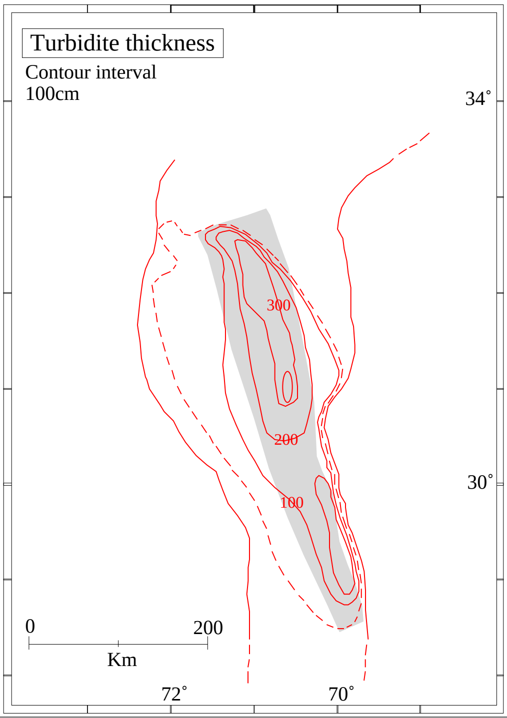
$$c = 3$$

$$x_r = 3\left(g'_o\, A^3 / V_s^2\right)^{1/5}$$

7.11

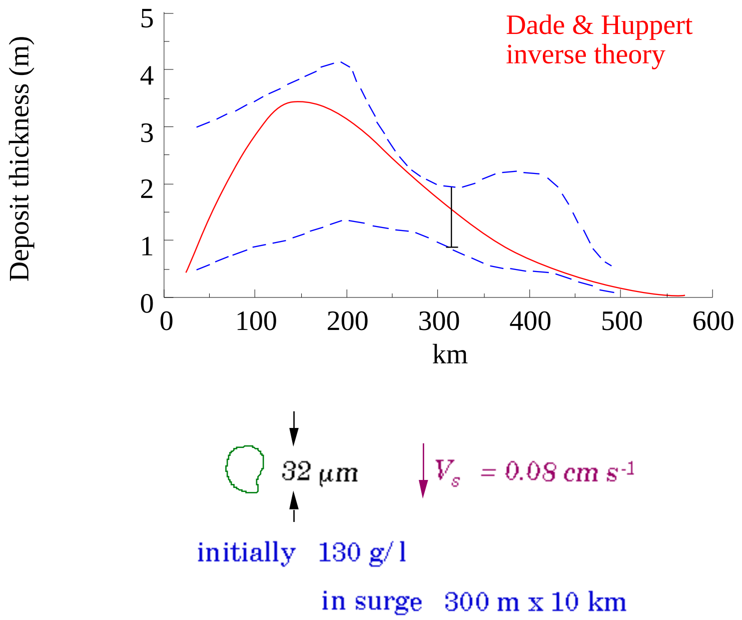


7.12



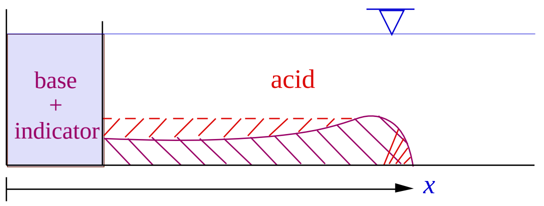
Turbidite thickness
Contour interval
100cm
34˚
300
200
100
30˚
0
200
Km
72˚
70˚

7.13



Deposit thickness (m)
5
4
3
2
1
0
0
100
200
300
400
500
600
km
Dade & Huppert
inverse theory
32 μm
V_s = 0.08 cm s-1
initially 130 g/l
in surge 300 m x 10 km

7.14



$$r = \frac{\text{volume of ambient entrained}}{\text{initial volume}} \equiv 0 \quad = \begin{cases} 0 & \text{initially } (x=0) \\ \infty & \text{for long currents } (x \to \infty) \end{cases}$$

$A_o \equiv L^2$ initial area

$g_o' = g\,\Delta\rho_o/\bar{\rho} \equiv L\,T^{-2}$ initial reduced gravity

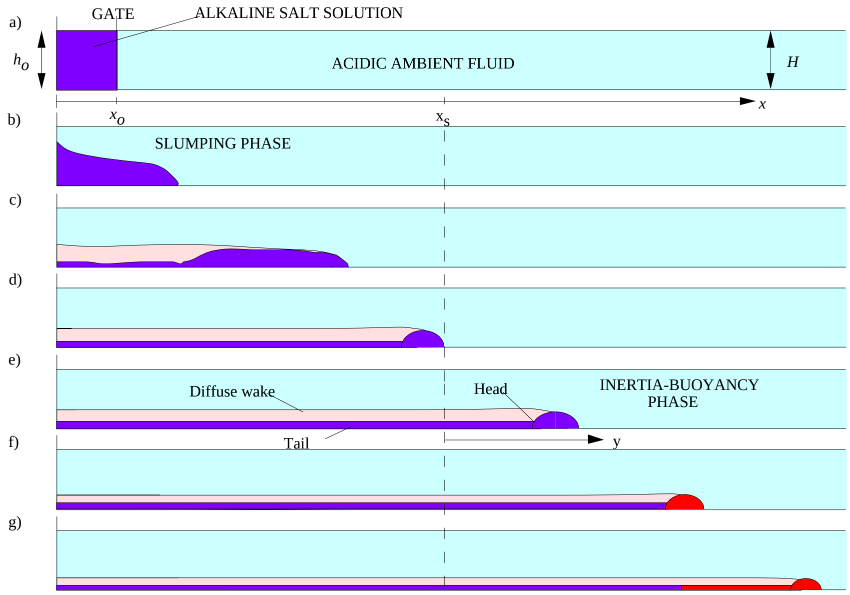
$x \equiv L$ length of currrent

$$r = f\left(\eta \equiv x/A_o^{1/2}\right) \text{ independent of } g_o' \quad = (1 - 0.035\eta)^{-2.26} - 1$$

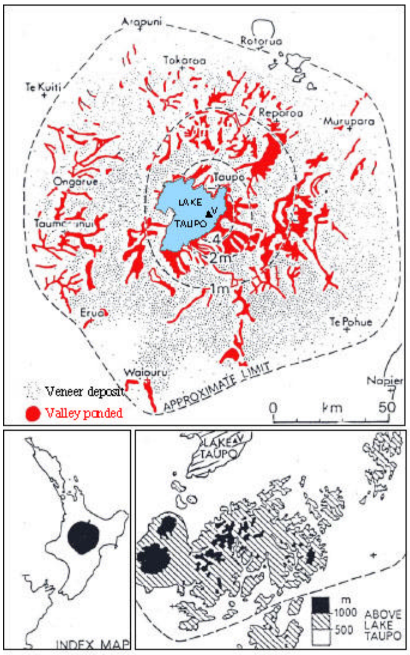
in agreement with experiments (see App.B)

(Hallworth, Phillips, Huppert & Sparks, 1993,1995)

7.15

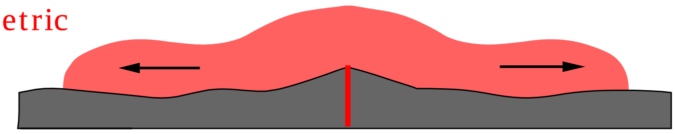


7.16



7.17

Axisymmetric



Q	volumetric flow rate	$L^3 T^{-1}$
V	mean fall speed of deposit	$L T^{-1}$
g'_o	initial reduced gravity of particle suspension	$L T^{-2}$

$$r_\infty \approx (Q/\pi V)^{1/2}$$

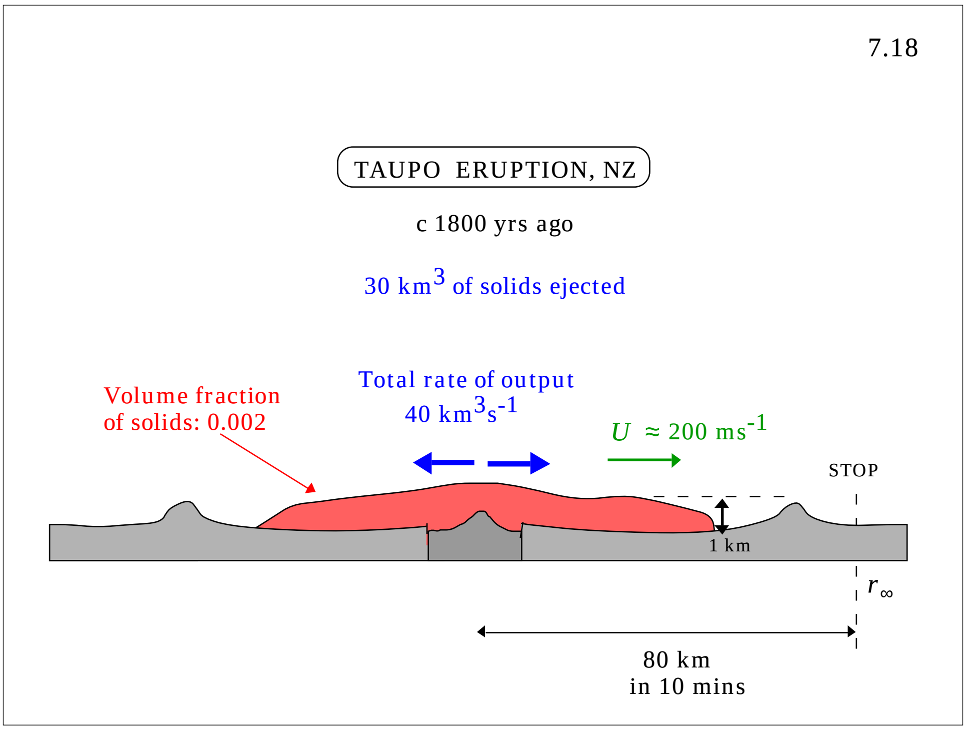
$$t_\infty \approx (Q/g'_o V)^{1/3}$$

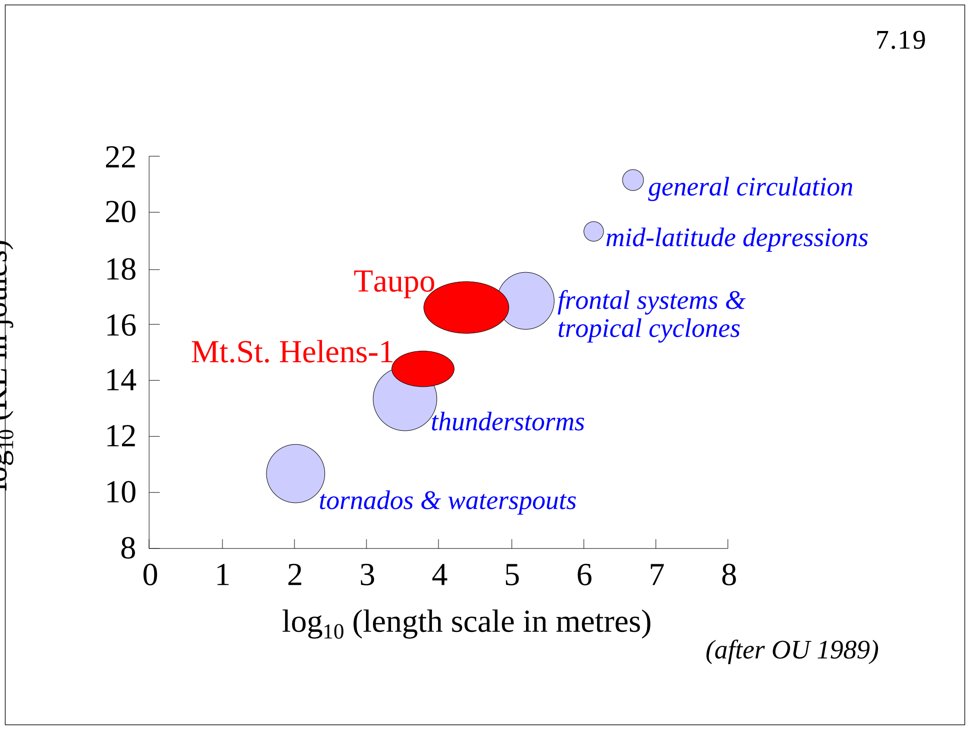
$$U \approx \left(Q V g_o'^{1/2}\right)^{1/6}$$

$$h \approx (Q/2\pi) r_\infty U$$

where

- r_∞ the final run-out distance
- t_∞ the time taken to reach there
- U a typical velocity
- h a typical thickness


7.18
TAUPO ERUPTION, NZ
c 1800 yrs ago
30 km^3 of solids ejected
Volume fraction of solids: 0.002
Total rate of output 40 km^{3}s^{-1}
U ≈ 200 ms^{-1}
STOP
1 km
r_∞
80 km in 10 mins


7.19
22
20
18
16
14
12
10
8
0
1
2
3
4
5
6
7
8
log$_{10}$ (KE in joules)
log$_{10}$ (length scale in metres)
general circulation
mid-latitude depressions
Taupo
frontal systems & tropical cyclones
Mt.St. Helens-1
thunderstorms
tornados & waterspouts
(after OU 1989)

CONCLUSIONS

7.20

- Bulk density, influenced by both temperature and particle concentration, plays a major role
- Models for the propagation of (thin) particle-driven currents on a horizontal surface can be derived and solved
- The predictions of these models are in good agreement with experimental data
- The results, and in particular the suggested scaling laws, will be very useful to geologists in their interpretation of some turbidite deposits
- Fluid entrainment from the ambient can be accounted for (in some cases)
- We have also investigated propagation down slopes, in rotating systems and over porous media, { and / but } there are further interesting challenges.

7.21

Lecture 7. Particle-driven Flows

Bonnecaze, R.T., Huppert, H.E. and Lister, J.R., 1993 Particle-driven gravity currents, *J. Fluid Mech.*, **250**, 339-369.

Bonnecaze, R.T., Hallworth, M.A., Huppert, H.E. and Lister, J.R.,1995 Axisymmetric particle-driven gravity currents, *J. Fluid Mech.*, **294**, 93-121

Dade, W.B. and Huppert, H.E.,1994 Predicting the geometry of deep-sea turbidites, *Geology* **22**, 645-648.

Dade, W.B., Lister,R. J.R. and Huppert, H.E., 1994 Fine-sediment deposition from gravity surges on uniform slopes, *J. Sed. Res.*, **A64**, 423-432

Dade, W.B. and Huppert, H.E., 1996 Emplacement of the Taupo ignimbrite by a dilute turbulent flow, *Nature*, **381**, 509-512 with News and Views article

Hallworth, M.A., Phillips, J., Huppert, H.E. and Sparks, R.S.J., 1993 Entrainment in turbulent gravity currents, *Nature*, **362**, 829-831.

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Huppert, H.E. 1998 Quantitative modelling of granular suspension flows, *Phil. Trans. Roy. Soc.*, **356**, 2471-2496.

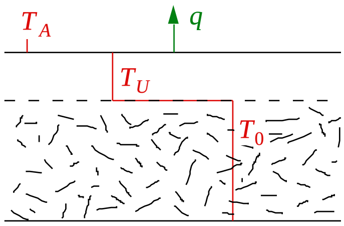
Koyaguchi, T., Hallworth, M.A. and Huppert, H.E., 1993 An experimental study on the effects of phenocrysts on convection in magmas, *J. Volcanol. and Geotherm. Res.*, **55**, 15-32.

Koyaguchi, T., Hallworth, M.A., Huppert, H.E. and Sparks, R.S.J., 1990 Sedimentation of particles from a convecting fluid, *Nature*, **343**, 447-450.

Martin, D. and Nokes, R., 1988 Crystal settling in a vigorously convecting magma chamber, *Nature*, **332**, 534-536.

APPENDIX A: COOLING FROM ABOVE

7.A1



$$\frac{d}{dt}\left[vt\left(T_U - T_0\right)\right] = \frac{q}{\rho c} = \beta\left(T_U - T_A\right)^{4/3}$$

$$T_U = T_0 \quad (t = 0) \qquad T_L \equiv T_0 \quad (\; t)$$

No solution to initial value problem unless $q \to 0$ as $t \to 0$, so that infinitesimal amount of heat extracted from infinitesimal layer.

$$t > 0: \quad T_U \sim T_0 + \beta\left(T_U - T_A\right)^{4/3} \quad = T_* \quad \text{with} \quad T_A < T_* < T_0$$

$\rho_U\left(T_*\right) > \rho_L\left(T_0, C\right)$ continual overturn

$$N = N_0 e^{-v_s t/H}$$

$\rho_U\left(T_*\right) < \rho_L\left(T_0, C\right)$ never overturns

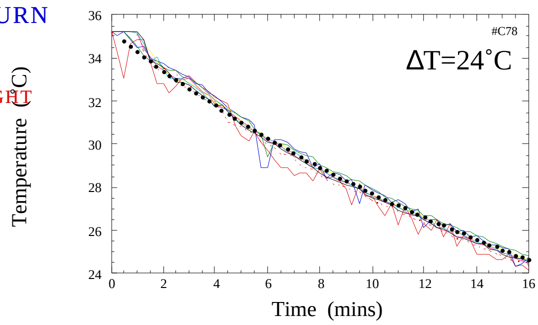
$$N = N_0\left(1 - v_s t/H\right)$$

7.A2

a) CONTINUAL OVERTURN

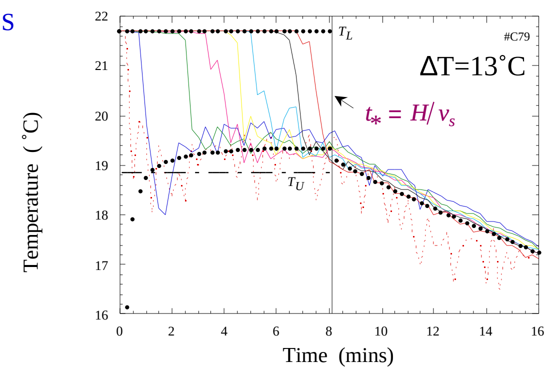
TEMP INDEPENDENT OF HEIGHT

LAYER SLOWLY COOLS

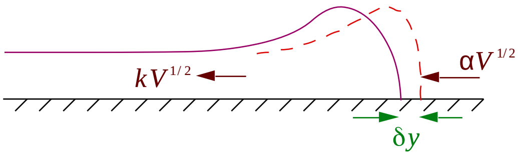


b) NEVER OVERTURNS

TEMP CONSTANT
(BUT DIFFERENT)
IN BOTHLAYERS



APPENDIX B: ANALYSIS OF ENTRAINMENT 7.B1



Givens : A_0 and $y = x - x_s$

Unknowns $V(y; A_o)$ volume per unit width of head

$$\delta V = \alpha V^{1/2} \delta y \quad - \quad k V^{1/2} \delta y$$

change in volume = input at front − output into body

(must be $V^{1/2}$ by dimensions)

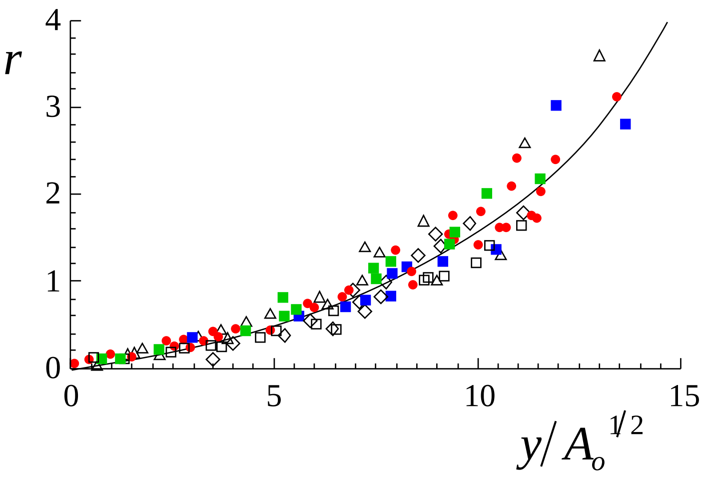
$$V(0) = A_o \qquad V/A_o = \left(1 - \tfrac{1}{2}\beta y / A_o^{1/2}\right)^2 \qquad \beta = k - \alpha$$

note that $V = 0$ at $y = 2 A_o^{1/2} / \beta \equiv y_c$

$$\left.\begin{aligned} \frac{dV_a}{dy} &= \alpha V^{1/2} - \frac{rk}{1+r} V^{1/2} \\ \frac{dV_b}{dy} &= \qquad\quad - \frac{k}{1+r} V^{1/2} \end{aligned}\right\} \qquad \begin{aligned} r &= V_a / V_b \\ V &= V_a + V_b \\ V_a(0) &= 0 \qquad V_b(0) = A_o \end{aligned}$$

$$r = \left(1 - \tfrac{1}{2}\beta y / A_o^{1/2}\right)^{-2\alpha/\beta} - 1 \qquad = \infty \qquad \text{at} \quad y = y_c$$

7.B2



$$r = \left(1 - 0.035\, y / A_o^{1/2}\right)^{-2.26} - 1$$