Lecture 1: Dynamical basics
Part II

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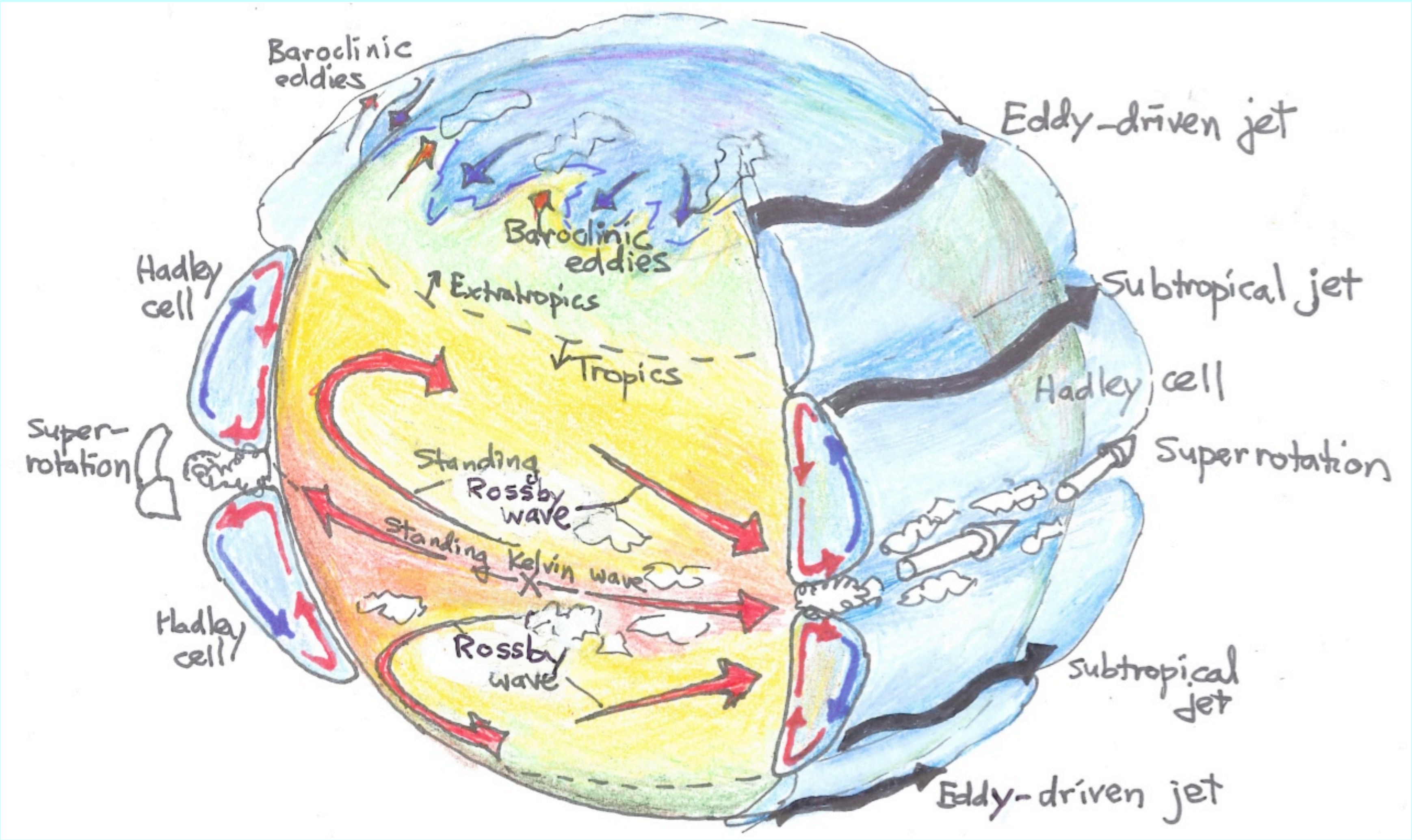
Recap of qualitative dynamical regimes:



Showman et al. (2013, "Atmospheric circulation of terrestrial exoplanets" in the book *Comparative Climatology of Terrestrial Planets*)

Key dynamical length scales

In the extratropics, the natural horizontal length scale associated with geostrophic adjustment (and other processes involving the interaction of gravity and rotation) is the Rossby deformation radius,

$$L_D = \frac{c}{f}$$

where c is the gravity wave speed. In the tropics the equivalent natural length scale, called the equatorial deformation radius, is

$$L_D = \left(\frac{c}{\beta}\right)^{1/2}$$

where $\beta = df/dy$ is the gradient of the Coriolis parameter with northward distance, y.

Extratropics

Ro << 1; dynamics is in geostrophic balance

Geostrophy enables large horizontal temperature contrasts (cf Charney 1963)

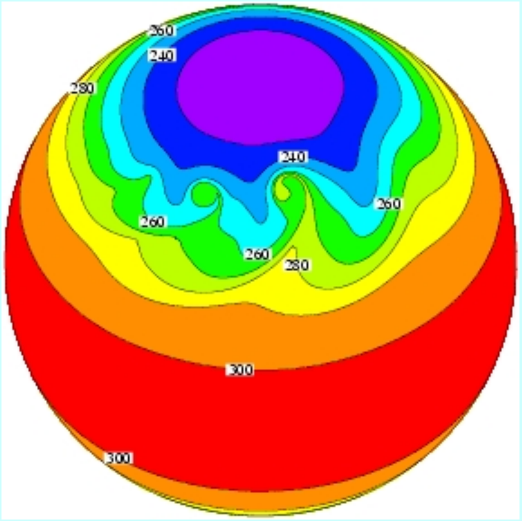
$$\frac{\delta\theta_{horiz}}{\theta} \sim \frac{fUL}{gD} \sim \frac{F}{Ro}$$

where $F=U^2/gD$ is a Froude number and $Ro=U/fL$ is the Rossby number. Here D is the depth of the system, U is wind speed, f is Coriolis parameter, L is horizontal lengthscale, and g is gravity. For Earth-like parameters, we obtain a temperature difference of ~0.01.

Large horizontal temperature contrasts and sloping isentropes imply that extratropics are generally baroclinically unstable. In analytic theory, the most unstable zonal wavelength is typically ~4L_D, with growth rates scaling with

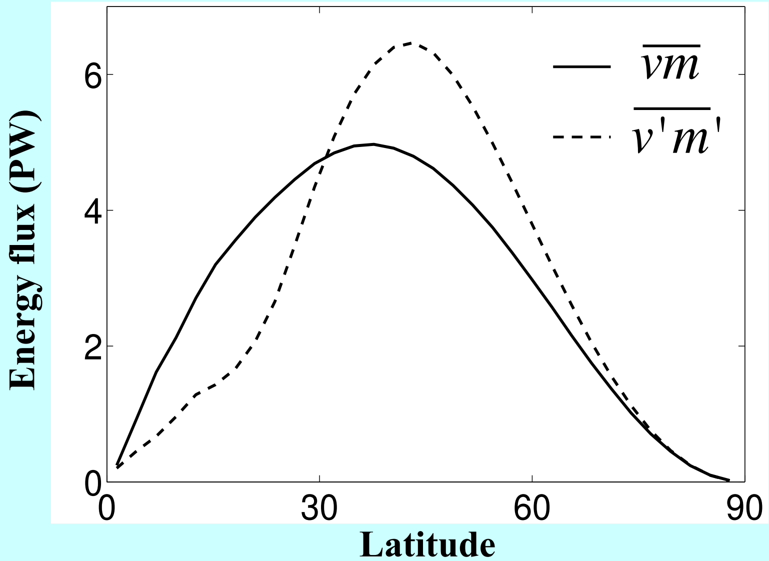
$$(f/N)\partial u/\partial z$$

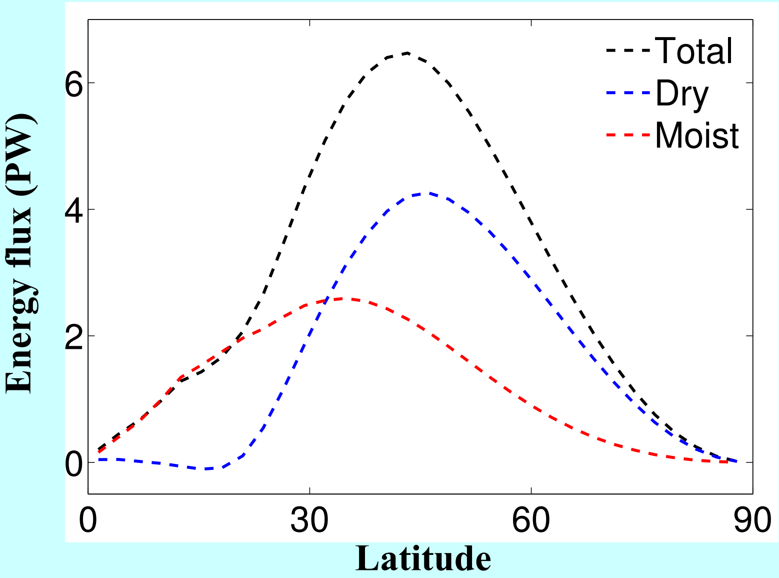
For Earth-like conditions, these imply length scales of ~4000 km and growth timescales of 3-5 days.



Baroclinic instabilities generate eddies that dominate much of the dynamics, controlling equator-pole heat fluxes, temperature contrasts, meridional mixing rates, vertical stratification, and jet formation

Meridional temperature distribution (Earth case)





m = moist static energy

$$= C_p T + gz + Lq$$

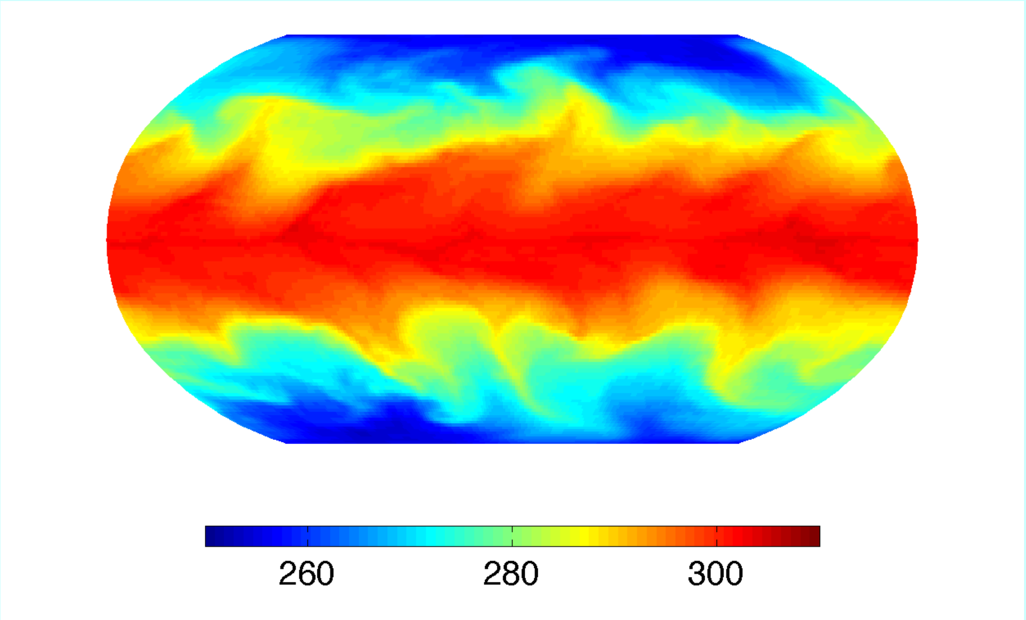
$$\overline{vm} = \bar{v}\bar{m} + \overline{v'm'}$$

T = **temperature**

z = **geopotential**

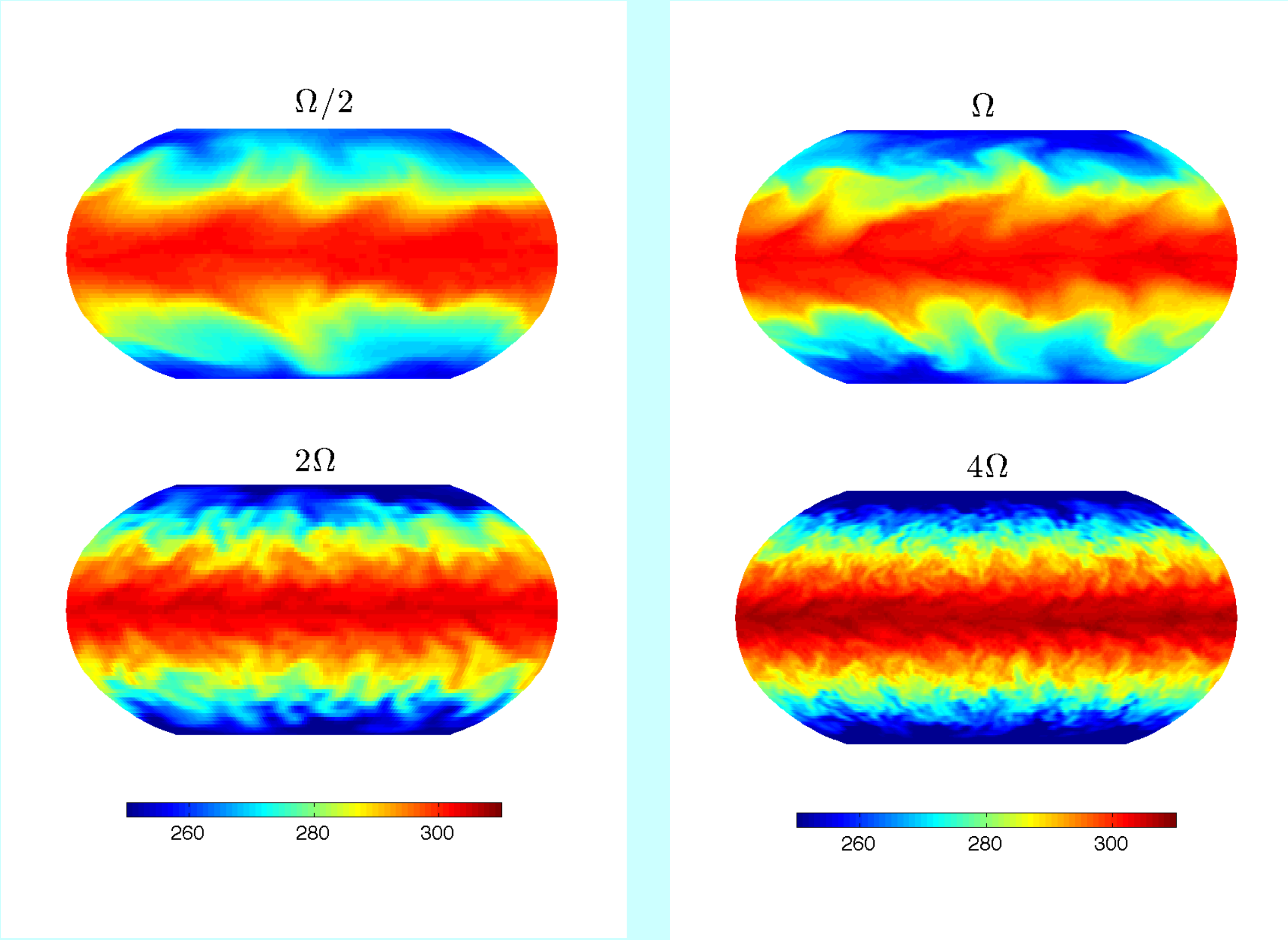
s = **specific humidity**

L = **latent heat of vaporization**



Kaspi & Showman (2015)

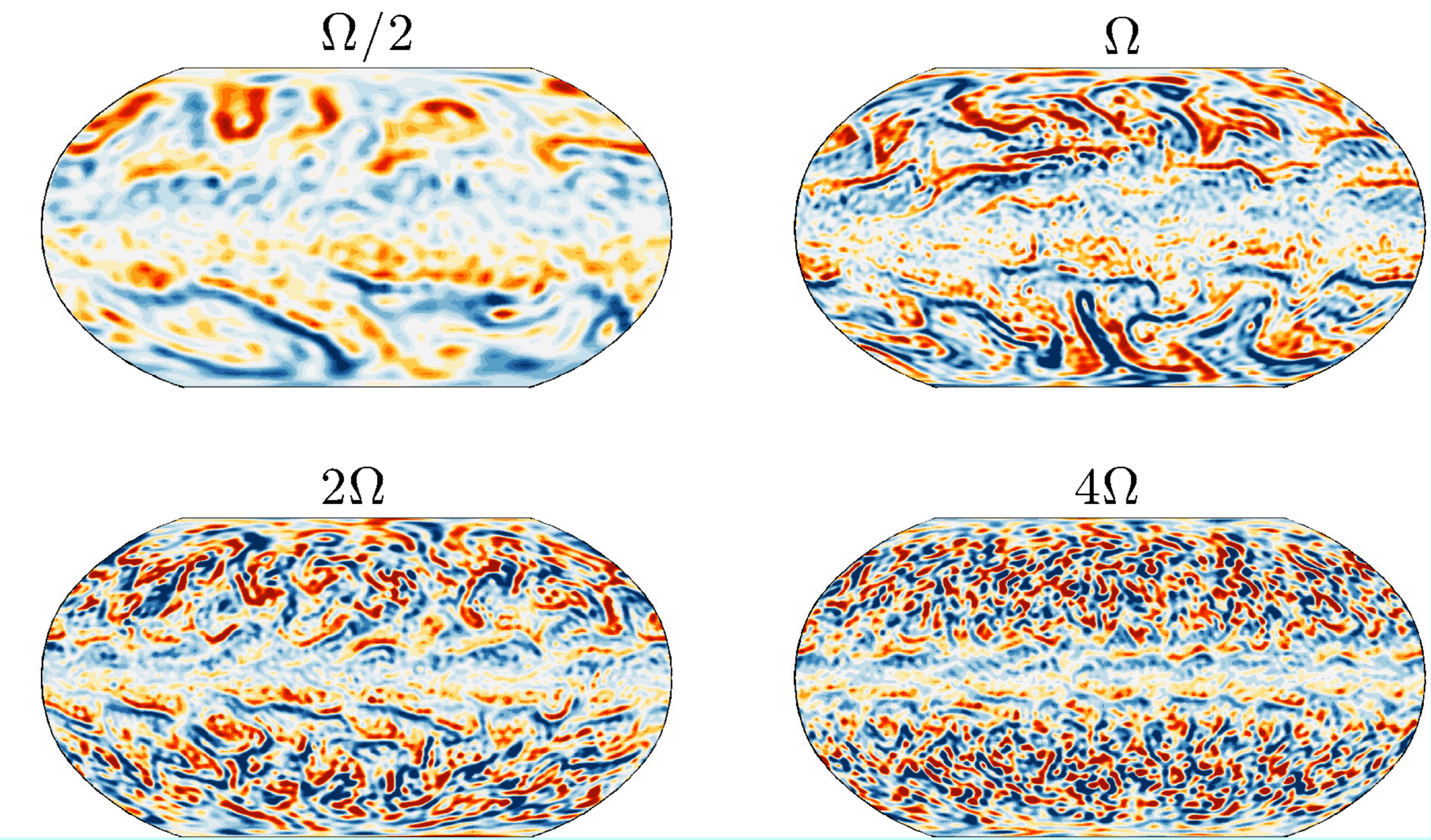
Effect of planetary rotation on baroclinic eddies



Kaspi & Showman (2015)

Eddy length scale decreases inversely with rotation rate, as expected from analytic theory. Note the greater equator-pole temperature differences at high rotation rate.

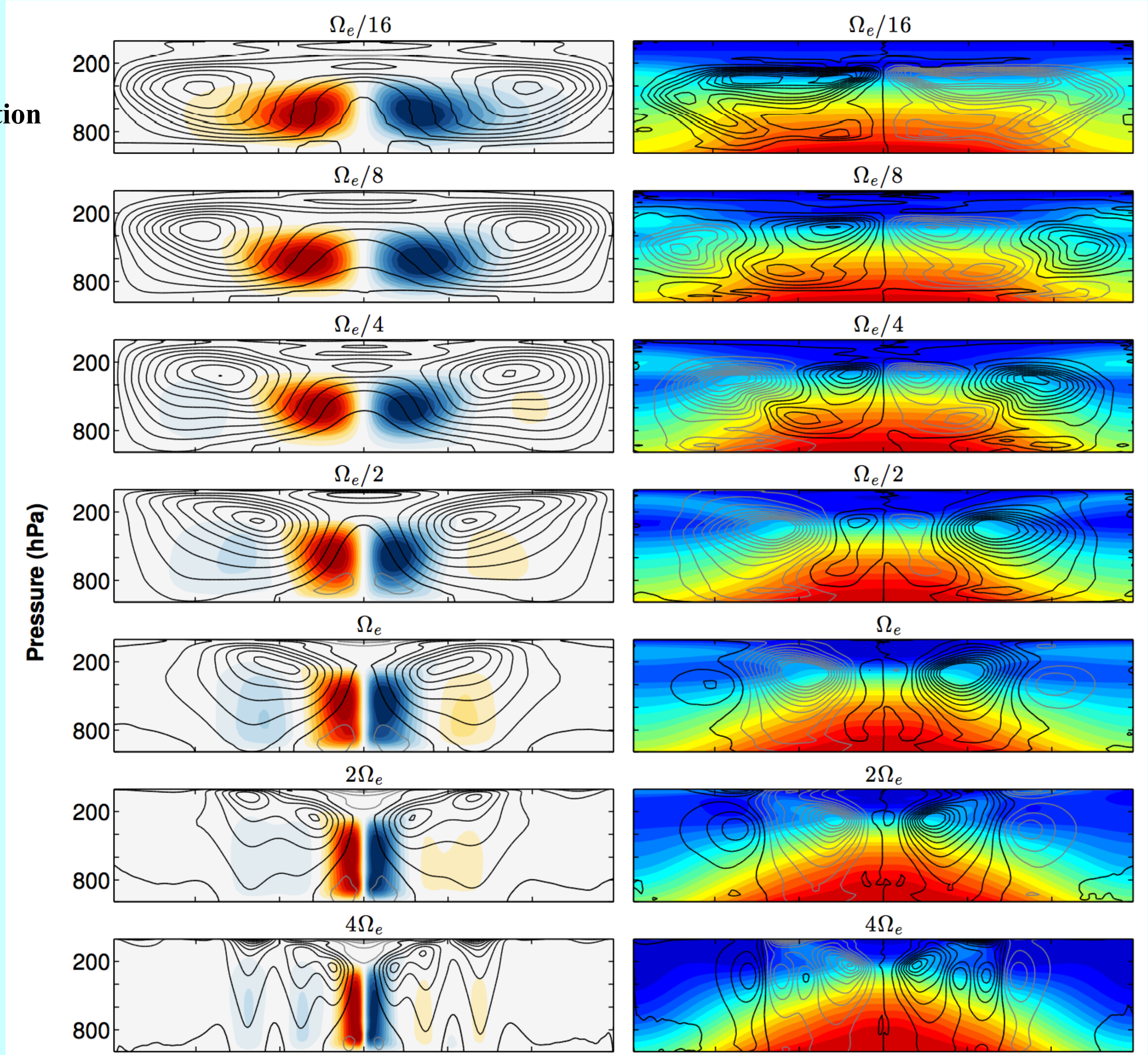
Mid-troposphere vorticity



Kaspi & Showman (2015)

Effect of planetary rotation

Streamfunction (color) zonal wind (contours)

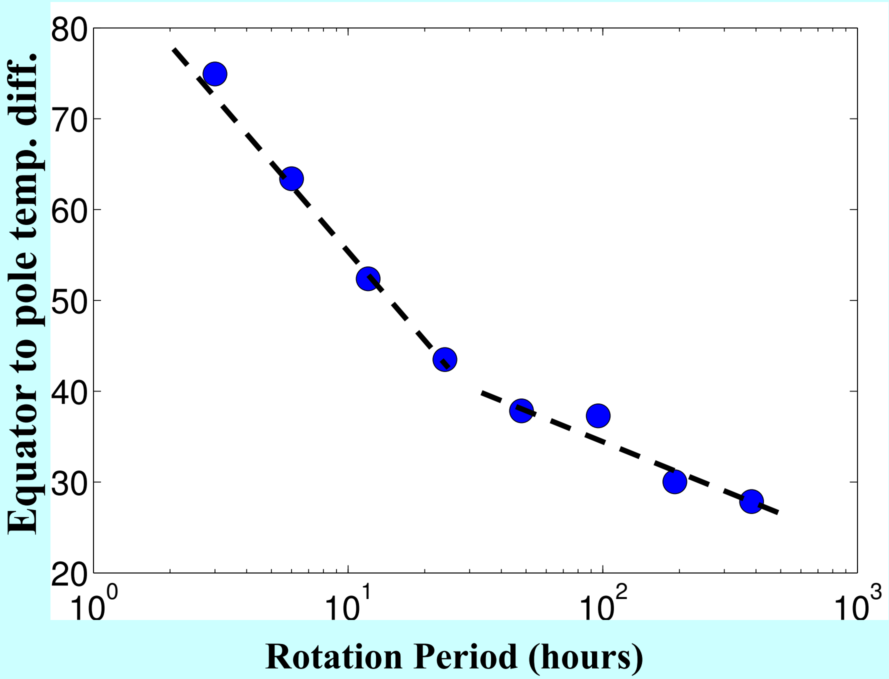


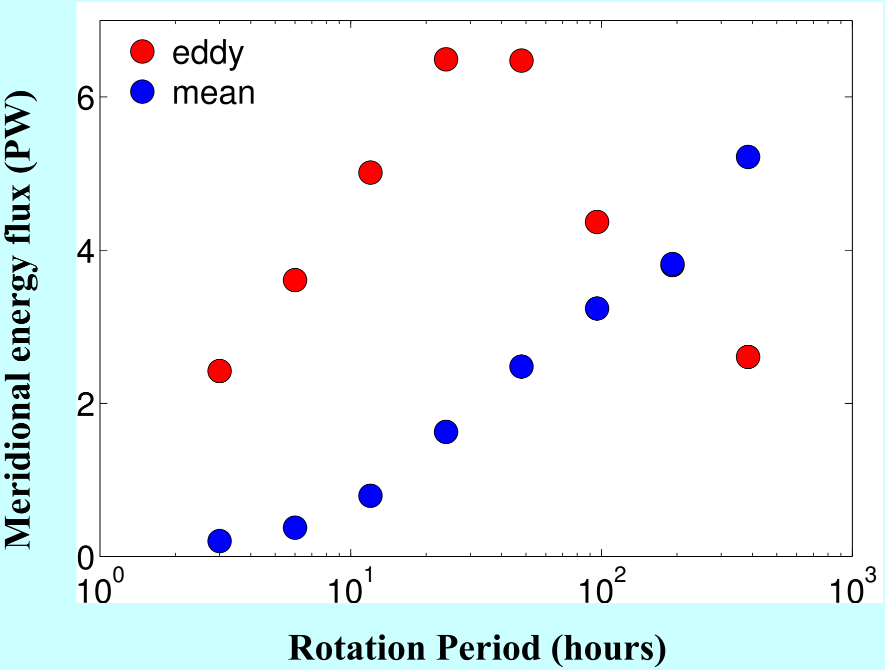
Temp (color) mom flux (contours)

Kaspi & Showman (2015)

Effect of planetary rotation

The smaller eddies in the more rapidly rotating models are less efficient at transporting thermal energy, contributing to greater equator-to-pole temperature difference at fast rotation rate.

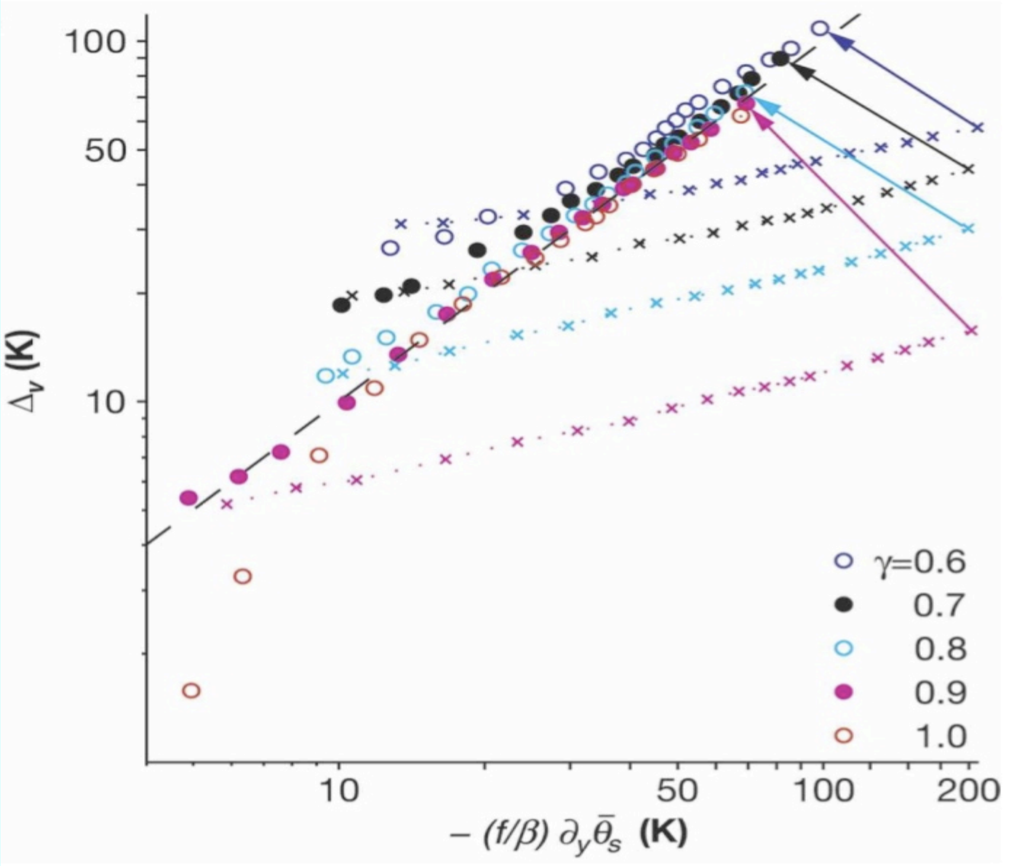




Kaspi & Showman (2015)

Extratropics: Role of baroclinic eddies in controlling thermal structure

Some numerical evidence suggests that baroclinic instabilities cause the extratropics to adjust to a state where isentropes slope by a scale height over a planetary radius (implying that vertical stratification scales with meridional temperature gradient). This could explain this property of Earth's extratropics. More on this in the lecture on jets.



Vertical stratification

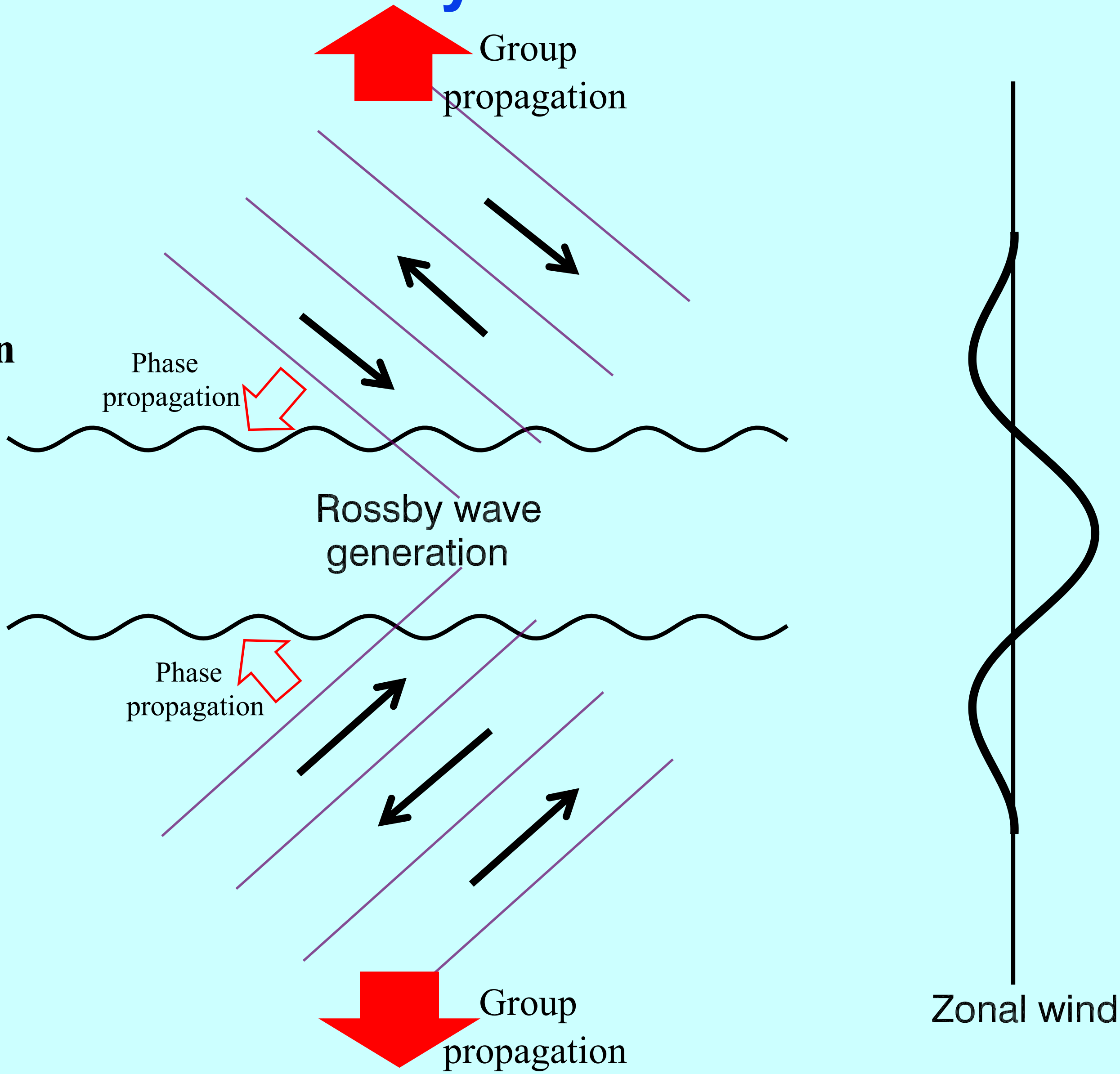
Meridional temperature gradient

Schneider & Walker (2006)

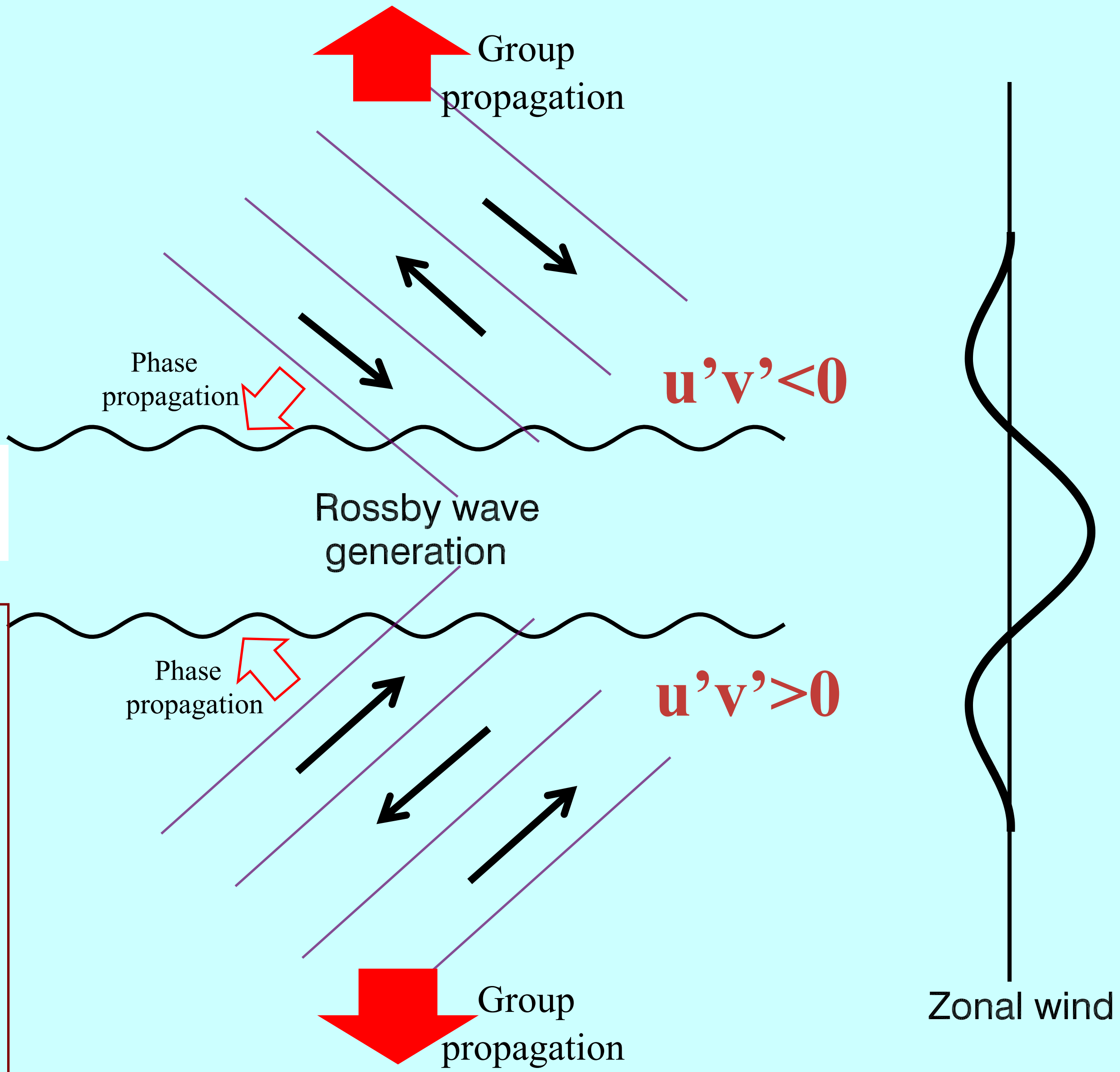
Rossby waves

Mechanisms of extratropical jet formation: role of Rossby waves

In the extratropics, regions of Rossby wave generation correspond to eastward eddy-driven jets. Regions of Rossby wave damping correspond to westward flow.



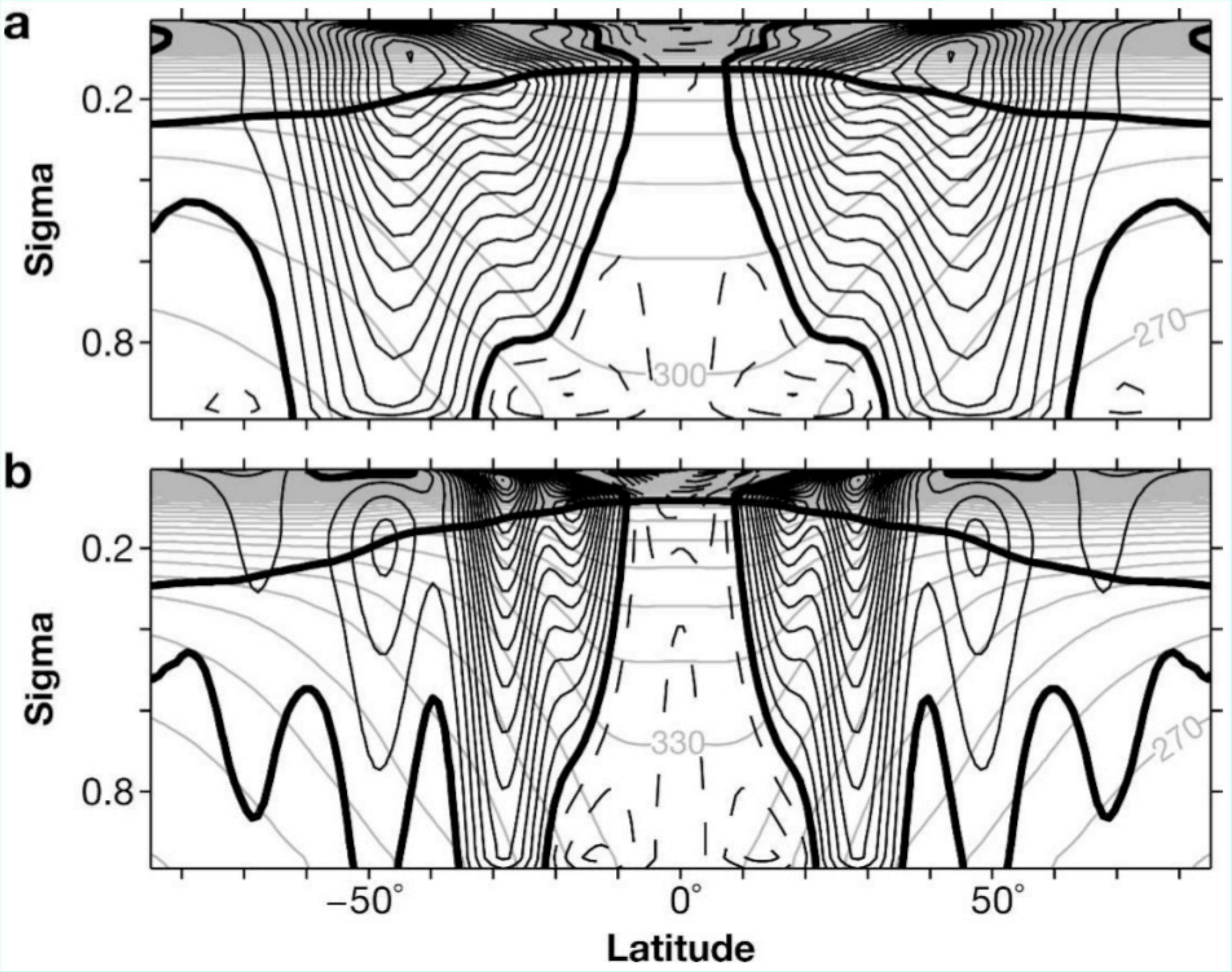
Mechanisms of jet formation: role of Rossby waves



$$\frac{\partial \overline{u}}{\partial t} = -\frac{\partial(\overline{u'v'})}{\partial y} - \frac{\overline{u}}{\tau_{\mathrm{drag}}}$$

The eddy acceleration term is positive (eastward) in the region of Rossby wave generation, and negative (westward) in the region of Rossby wave damping/breaking.

Zonal jets



Schneider & Walker (2006)

Tropics

Ro ~ 1; dynamics is inherently ageostrophic

Horizontal temperature contrasts tend to be small (cf Charney 1963)

$$\frac{\delta\theta_{horiz}}{\theta} \sim \frac{U^2}{gD} \sim F$$

which is significantly smaller than in the extratropical case. Inserting Earth parameters gives ~0.001.

Baroclinic instability less important or negligible (compared to the extratropics)

Temperature structure regulated by Hadley circulations and wave adjustment, contributing to the relatively small horizontal temperature differences, the so-called "weak temperature gradient" or WTG regime.

Hadley circulation

Regulates the thermal structure in the tropics. Exerts a significant effect on the mean climate. Although the real Hadley cell has strong 3D structure, it can be idealized as a 2D circulation—unlike the case of heat transport by baroclinic eddies in the extratropics.

All the terrestrial planets with thick atmospheres—Earth, Mars, Venus, Titan—have Hadley circulations.

Planetary rotation exerts strong control over the Hadley circulation. It's useful to think about the limit where the upper branch conserves angular momentum about the rotation axis,

$$m = (\Omega a \cos\phi + u) a \cos\phi,$$

where *a* is planetary radius. If the ascending branch is at the equator and exhibits zero zonal wind, then the zonal wind in the upper (poleward flowing) branch is

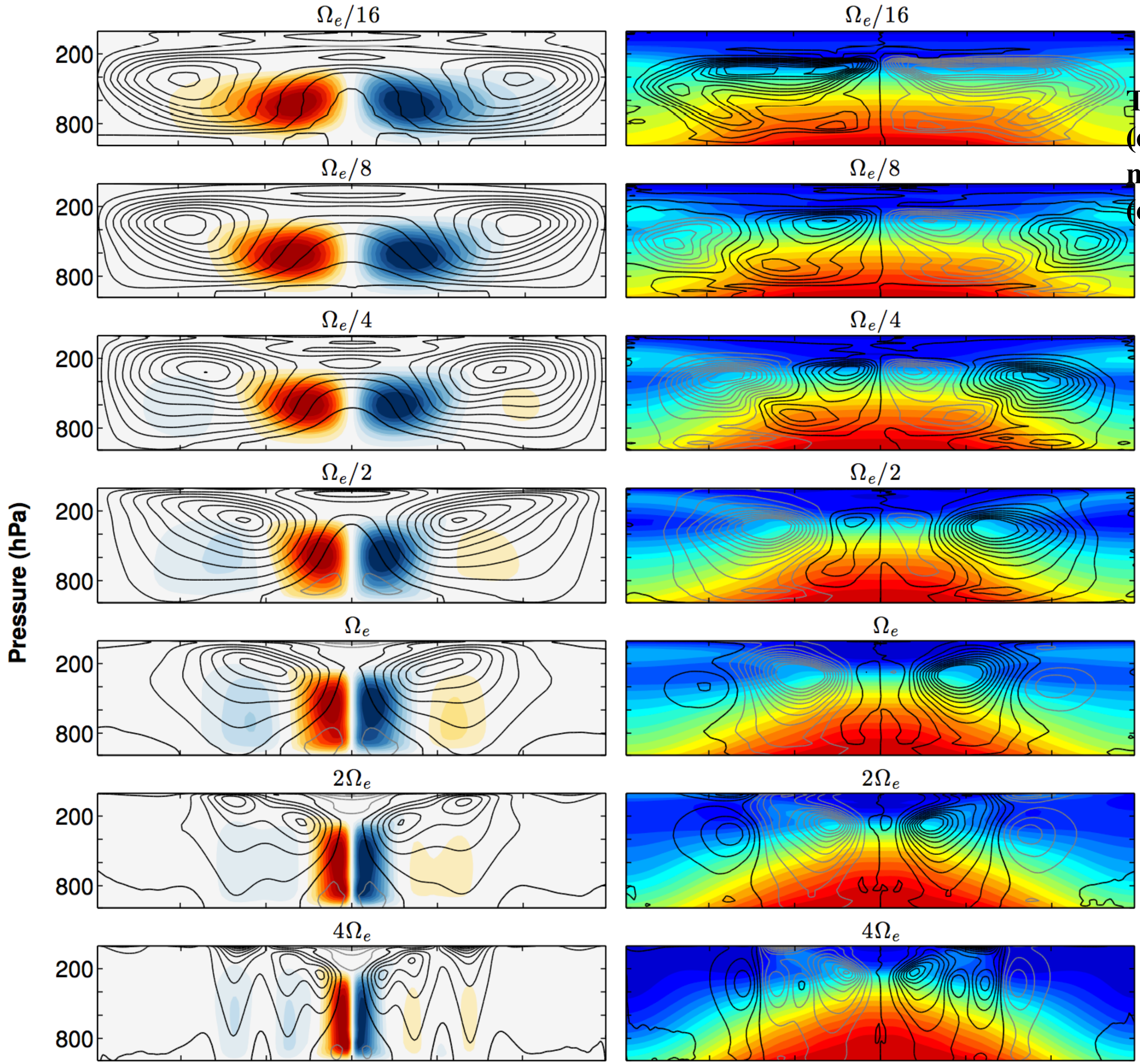
$$u = \Omega a \frac{\sin^2\phi}{\cos\phi}$$

For Earth conditions, this yields wind speeds of 134 m/s, 1000 m/s, and infinite at latitudes of 30^o, 67^o, and the poles. This is of course impossible, and implies that rotation, if sufficiently strong, will confine the Hadley circulation to low latitudes.

This eastward upper tropospheric wind, which peaks near the outer edge of the Hadley cell, is the *subtropical jet*.

Effect of planetary rotation

Streamfunction (color)
zonal wind (contours)



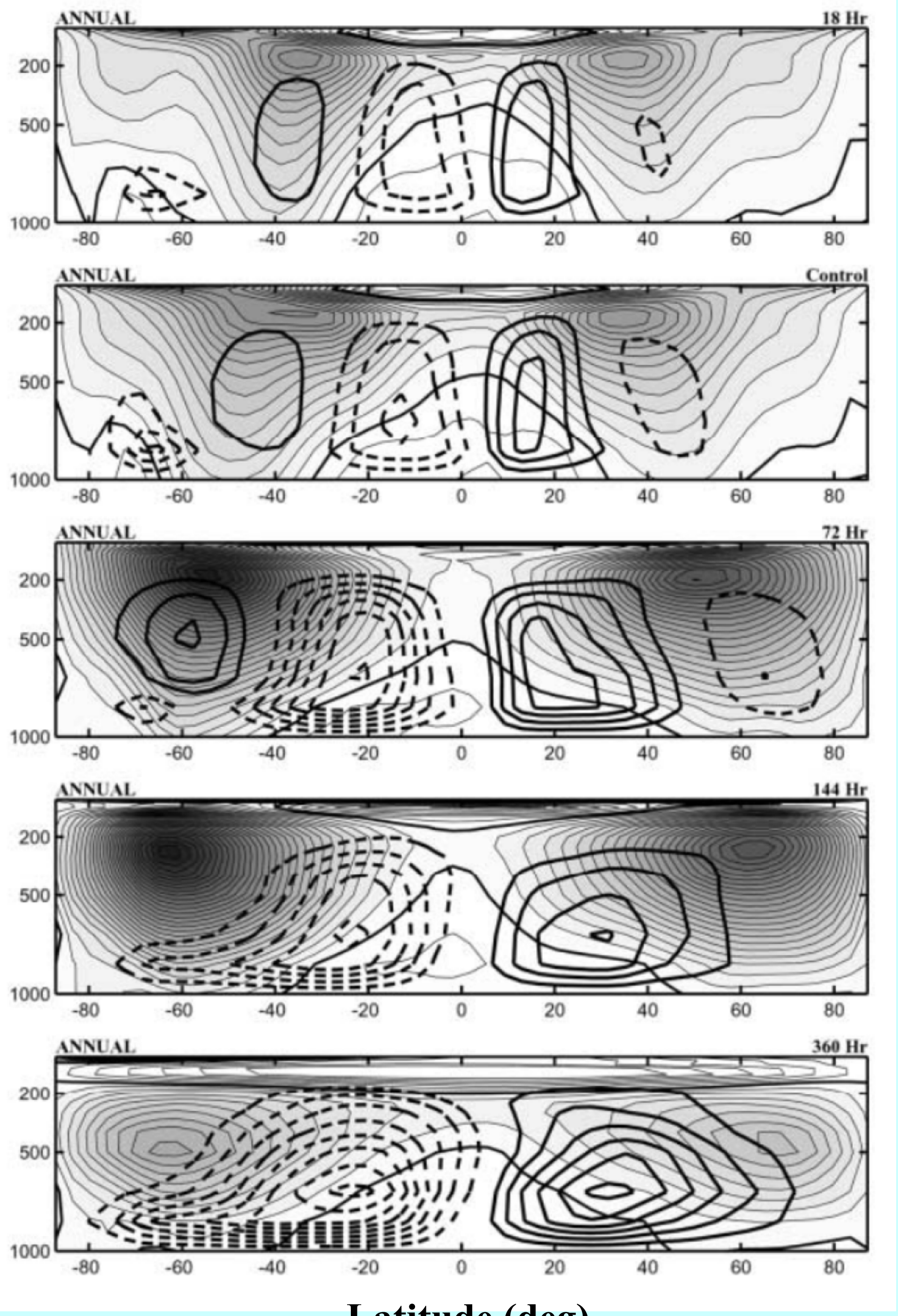
Temp (color)
mom flux (contours)

Note how width of Hadley cell increases with decreasing rotation rate, becoming nearly global at the slowest rotation rates.

Temperature gradients are relatively weak across most of the Hadley cell.

Kaspi & Showman (2015)

Increasing rotation period



Latitude (deg)

Navarra & Bocaletti (2002)

Hadley circulation

The Hadley circulation can exhibit different regimes depending on the extent to which the upper branch is angular-momentum conserving. Consider the zonal-mean zonal wind equation from 3D primitive equations, using pressure as a vertical coordinate:

$$\frac{\partial \overline{u}}{\partial t} = (f+\overline{\zeta})\overline{v} - \overline{\omega}\frac{\partial \overline{u}}{\partial p} - \frac{1}{a\cos^2\phi}\frac{\partial(\cos^2\phi\,\overline{u'v'})}{\partial \phi} - \frac{\partial(\overline{u'\omega'})}{\partial p}$$

where $\omega = dp/dt$ is the vertical velocity in pressure coordinates. Overbars and primes denote zonal means and deviations therefrom. Denote the eddy terms by $-S$ and consider the statistical steady state:

$$(f+\overline{\zeta})\overline{v} = \overline{\omega}\frac{\partial \overline{u}}{\partial p} + S$$

For Earth, the first term on the right side is not dominant, so that we can write (e.g., Held 2000, Walker & Schneider 2006):

$$(f+\overline{\zeta})\overline{v} = f(1 - Ro_H)\overline{v} \approx S$$

where $Ro_H = -\zeta/f$ is a Rossby number associated with the Hadley circulation.

Essentially, Ro_H is a measure of the strength of eddies on the Hadley cell, which exhibits different behaviors depending on whether Ro_H is large or small.

Hadley circulation

$$(f+\overline{\zeta})\overline{v} = f(1-Ro_H)\overline{v} \approx S$$

The Hadley circulation exhibits different behavior depending on whether Ro_H is large or small.

When eddy accelerations are negligible, then S=0, and for non-zero circulations the absolute vorticity must therefore be zero within the upper branch, i.e., $f+\zeta=0$, or, in other words, $Ro_H \rightarrow 1$. The definitions of relative vorticity and angular momentum imply that

$$f+\overline{\zeta} = \frac{1}{a^2\cos\phi}\frac{\partial\overline{m}}{\partial\phi}$$

A circulation with zero absolute vorticity therefore exhibits angular momentum that is constant with latitude. This is simply the angular-momentum conserving limit. The Hadley cell in this limit is thermally driven.

On the other hand, eddy accelerations are often important in shaping the Hadley circulation. If $Ro_H \ll 1$, then the zonal momentum balance is

$$f\overline{v} = S$$

which means that the strength of the Hadley circulation is solely controlled by the amplitude of the eddy acceleration (and not, at least directly, by thermal forcing).

Real Hadley circulations lie between these two extremes.

Hadley circulations

For Earth and Mars, the primary eddy effects result from absorption of equatorward-propagating Rossby waves that reach critical levels on the flanks of the subtropical jets. These waves break and cause a net westward torque, removing angular momentum.

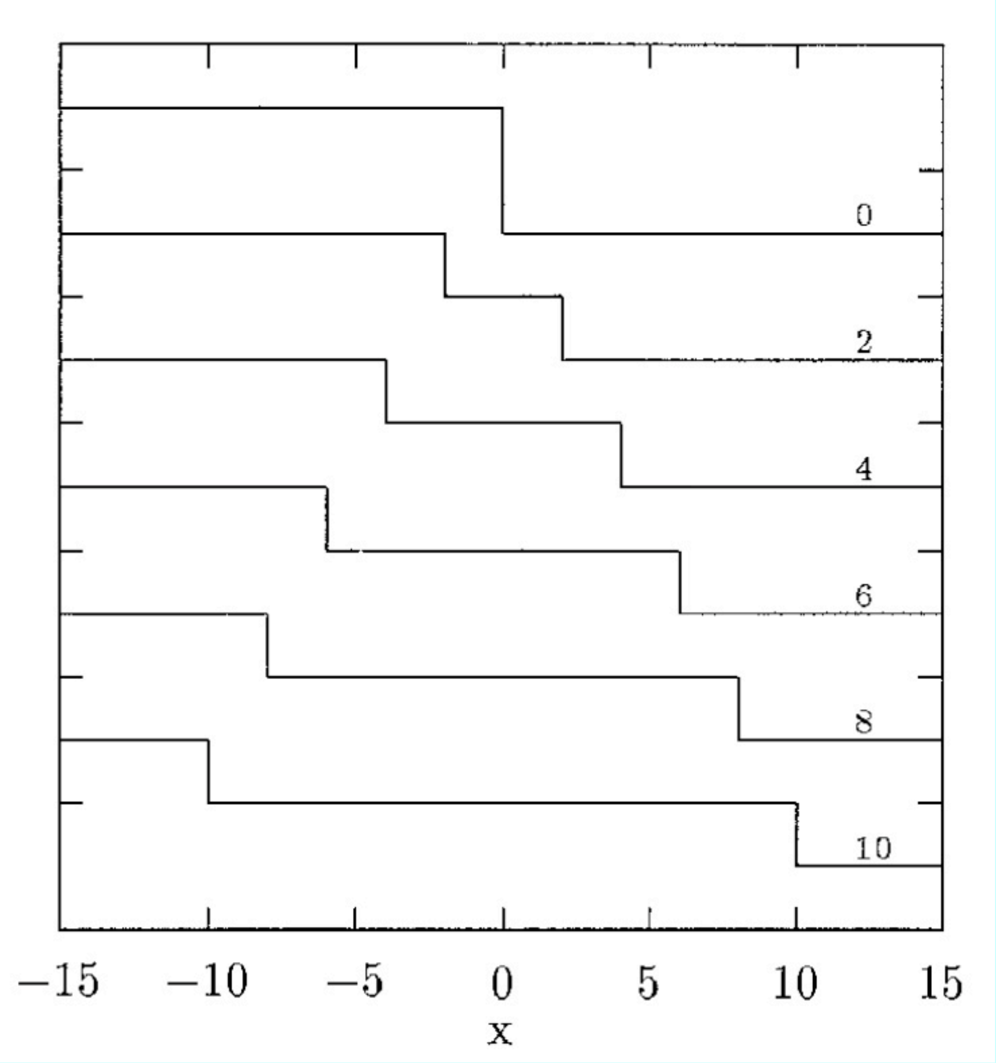
This implies that the angular momentum in the upper branch decreases with latitude away from the equator, and helps explain why the subtropical jet is a factor of several weaker than the angular-momentum conserving limit would suggest.

There is strong seasonality—if the rising branch is located off the equator, as occurs during solstice, then the so-called "winter cell" (the cell that crosses over the equator into the winter hemisphere) will have strongly westward winds near the equator, which tend to lack critical levels and is therefore relatively transparent to the waves.

Ro_H varies from ~0.3-0.4 in the equinoctal and summer cells to ~0.7-0.8 in the winter cell (Schneider & Bordoni 2008, Bordoni & Schneider 2008).

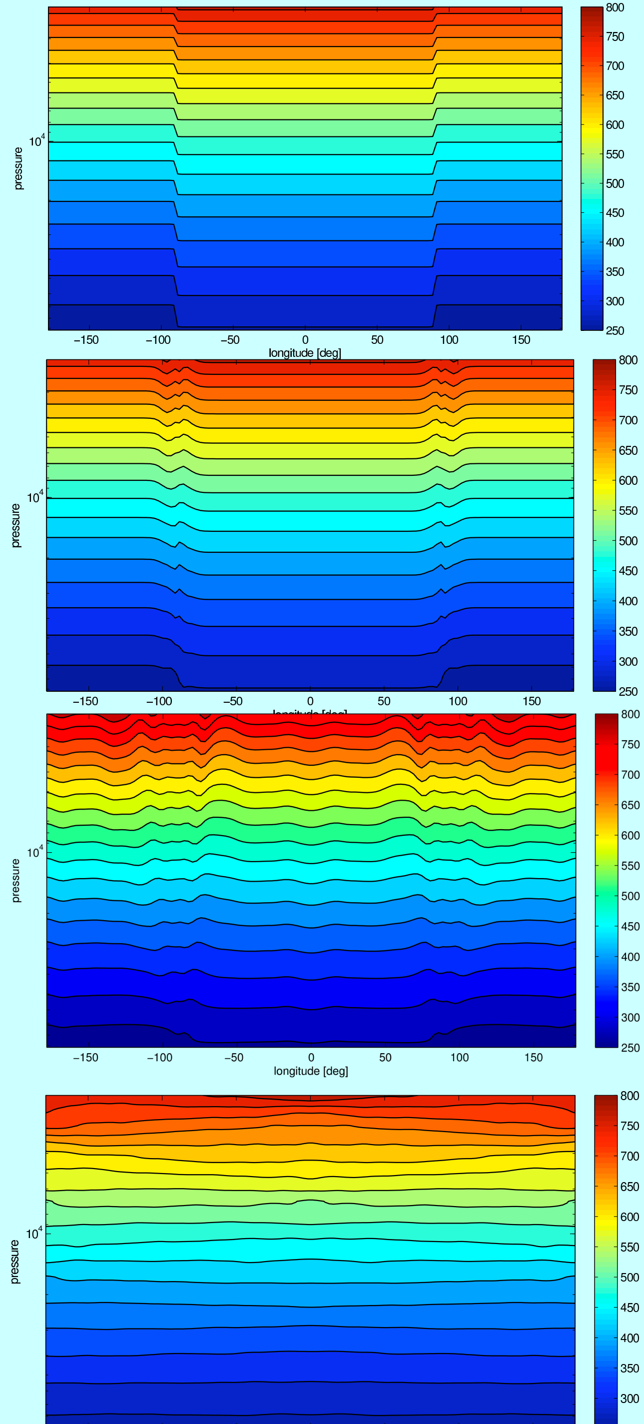
Wave adjustment

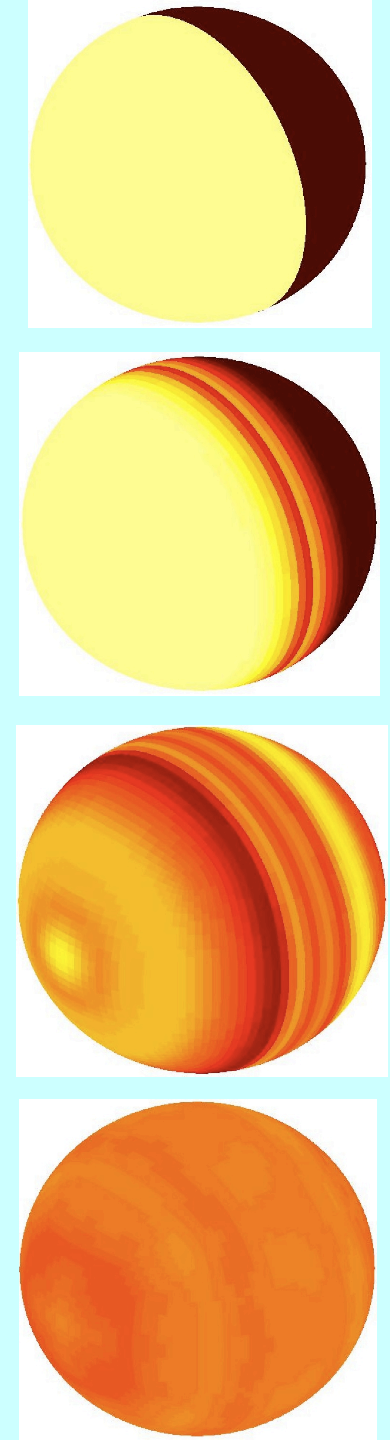
The "dam break" problem for the non-rotating case:



Kuo & Polvani (1997)

In a 3D tropical atmosphere, wave adjustment erases horizontal temperature differences





Showman et al. (2013, “Atmospheric circulation of terrestrial exoplanets” in the book *Comparative Climatology of Terrestrial Planets*)

Timescale arguments associated with adjustment

- **Generally one might crudely expect that if**

$$\tau_{damp} \geq \tau_{dyn} \implies \text{small fractional temperature differences}$$

$$\tau_{damp} \leq \tau_{dyn} \implies \text{large fractional temperature differences}$$

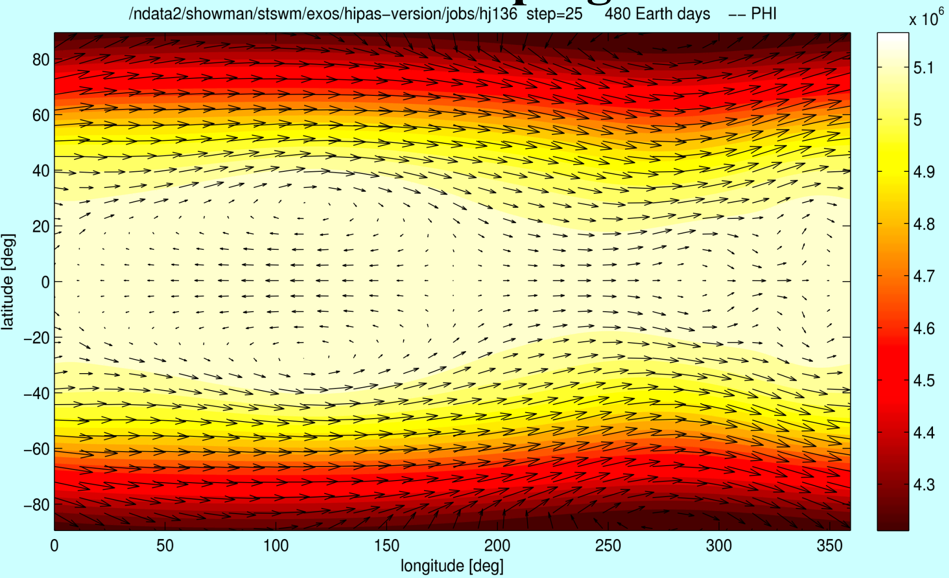
- **Relevant damping timescales include friction and radiative timescales. Dynamical timescales can be horizontal wave propagation timescales, although advection and rotation timescales may be relevant. The precise timescale comparison is thus probably more complex than shown above (e.g., see Komacek & Showman 2016).**

- **The adjustment timescale is often much shorter than the mixing timescale.**

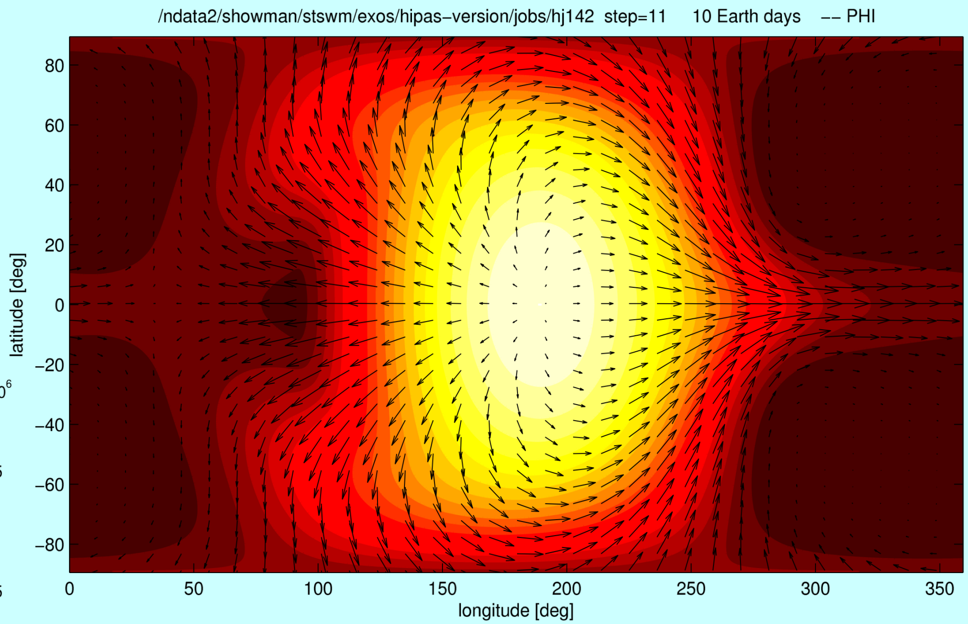
- **For synchronously rotating Earth-like planets, these arguments suggest large temperature differences if atmospheric pressure $\lesssim$ 0.1 bar. For hot Jupiters, they suggest large temperture differences if the planet is hot enough.**

This has important implications for atmospheric collapse on synchronously rotating planets (e.g., Joshi et al. 1997), and for explaining IR data for hot Jupiters.

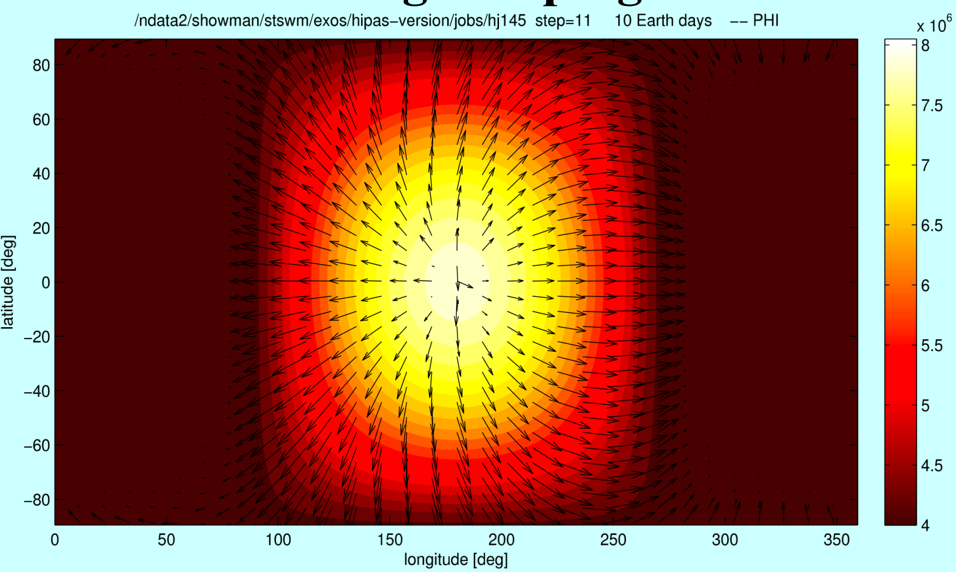
Weak damping



Moderate damping

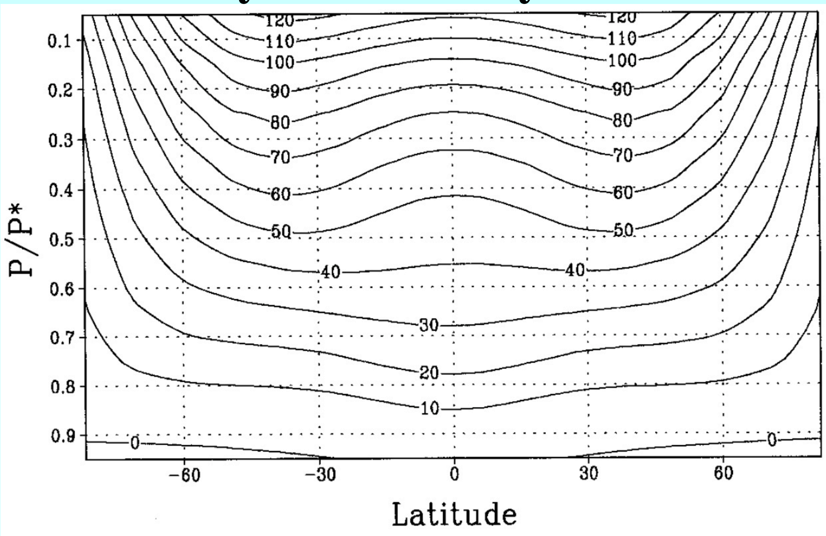


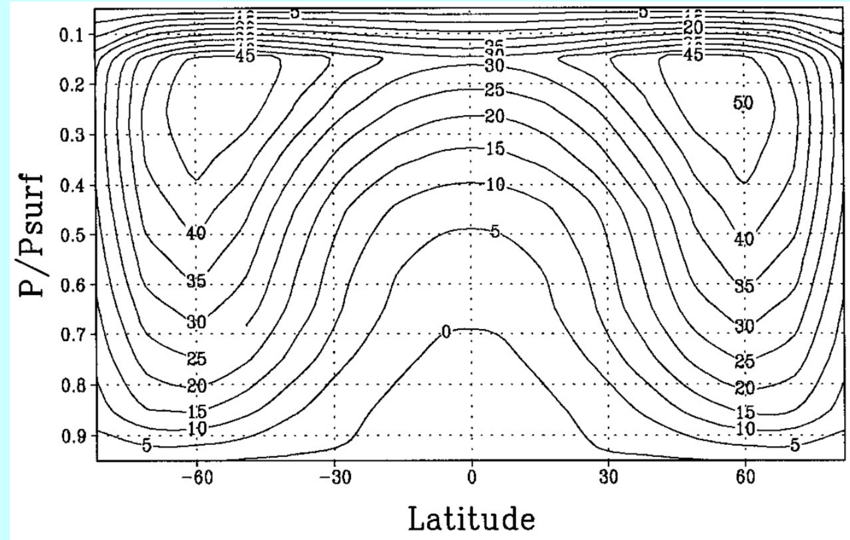
Strong damping



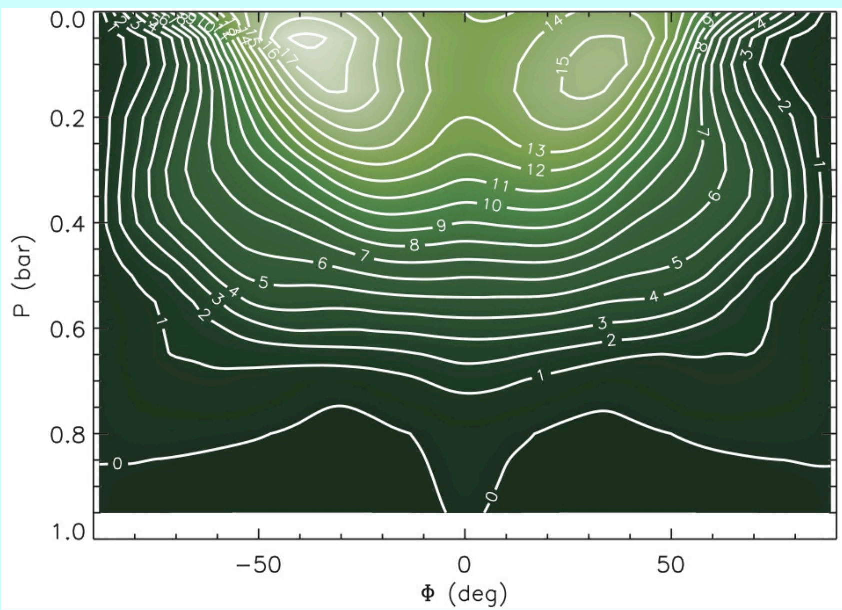
Equatorial superrotation is expected on tidally locked exoplanets

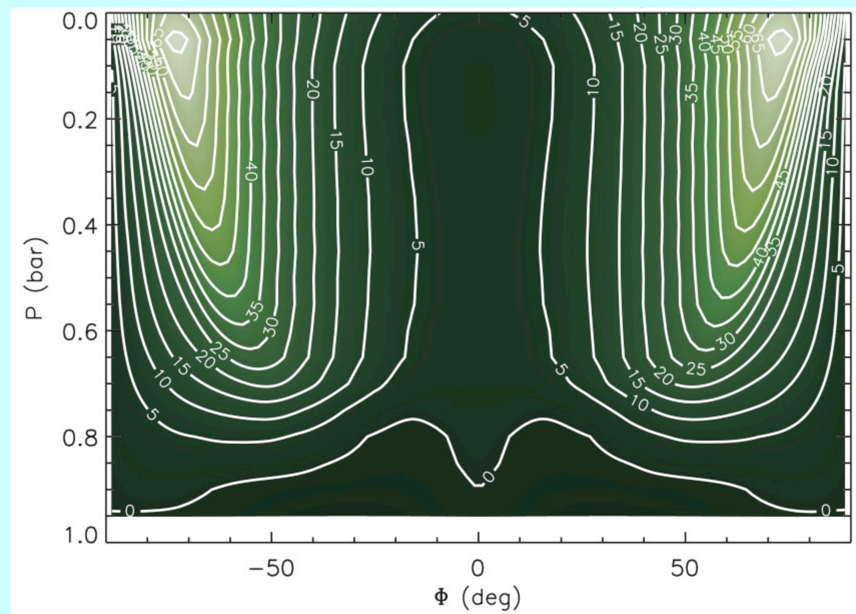
Synchronously locked | **Axisymmetric forcing**





Joshi et al. (1997)

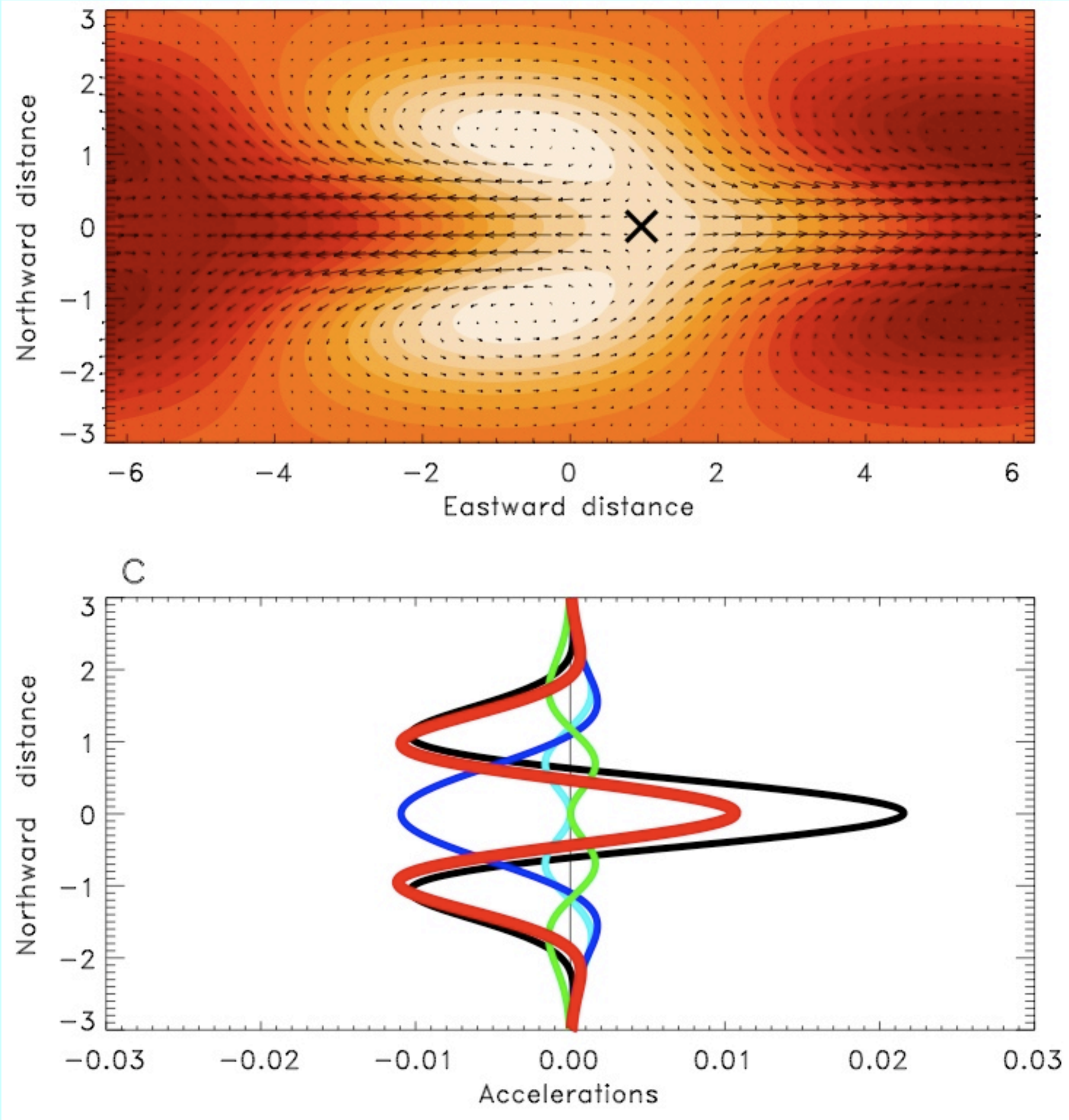




Heng & Vogt (2010)

See also Merlis & Schneider (2010), Wordsworth et al. (2011), Edson et al. (2011)

Showman & Polvani (2011) showed that the superrotation results from momentum transport by standing, planetary-scale waves driven by the day-night thermal forcing



Showman & Polvani (2 011, *ApJ* 738, 71)

Effect of circulation on global climate

Atmospheric circulation affects or controls the…

- **Existence and net climatic effect of clouds**

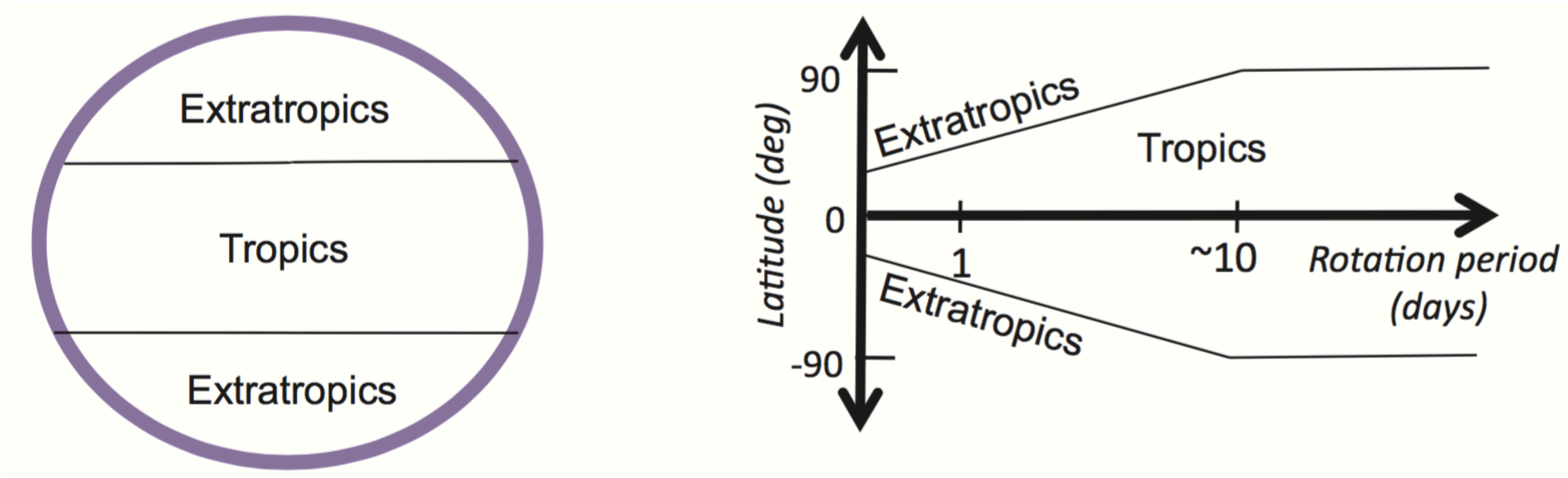
- **Heat fluxes and temperature contrasts between equator and pole (or day-night on tidally locked planets)**
  - **Susceptibility of atmosphere to collapse and/or global Snowball glaciation depends on atmospheric circulation**

- **Relative humidity and existence or absence of local dry regions, affecting transition to runaway greenhouse**

Planetary habitability and the width of the classical habitable zone depends on the atmospheric circulation

Regimes of atmospheric circulation

This shows how the latitudinal extent of the tropics and extratropics depend on rotation rate for a typical terrestrial planet. The globe on the left is for an Earth- or Mars-like world.



The tropics fills the entire globe for rotation periods greater than ~10 days for typical terrestrial planet parameters. Such planets are "all tropics" worlds. Titan and Venus are examples in our solar system. Many habitable-zone terrestrial exoplanets will also be in this regime.

Regimes of atmospheric circulation

Key to understanding atmospheric circulation is understanding the extent to which rotation dominates the dynamics. The importance of rotation is characterized by Rossby number

$$Ro = \frac{U}{fL}$$

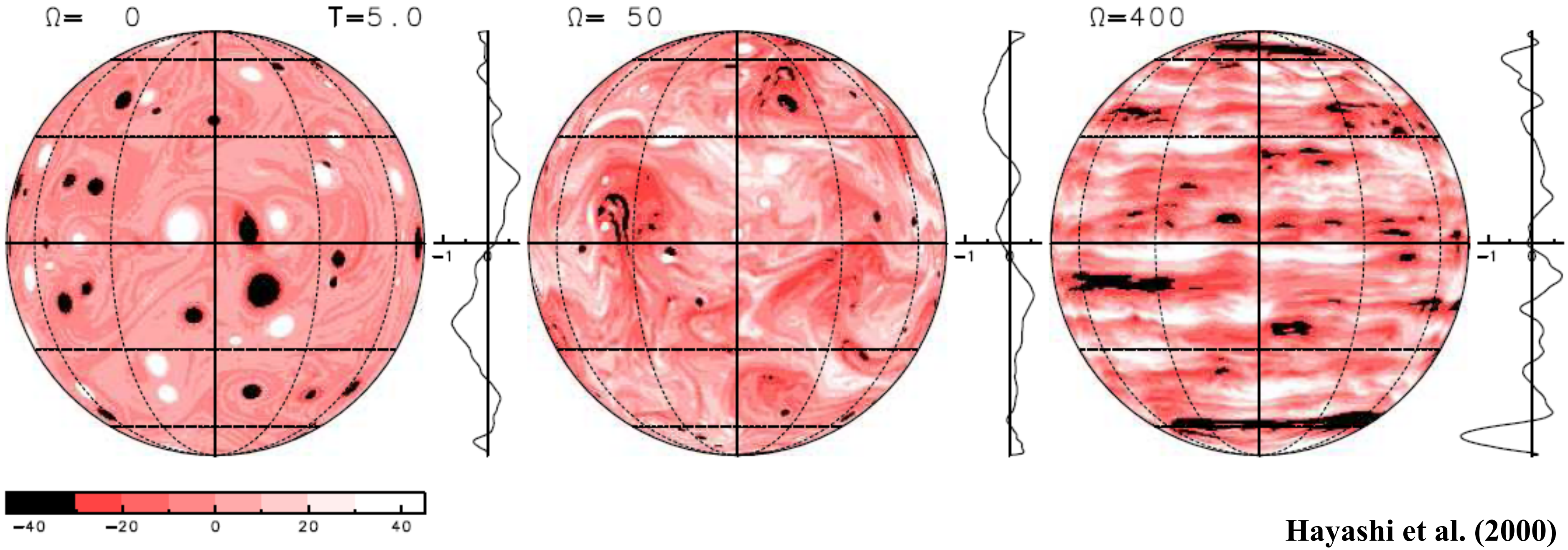
where U is characteristic wind speed, L is horizontal lengthscale, and f=2Ωsinϕ is the Coriolis parameter.

The dynamics at *Ro*<<1 differ substantially from those of *Ro*>1, leading to two regimes, which we define as follows:

Extratropics: **Regime of *Ro* << 1. Rotationally dominated; can support large horizontal temperature differences.**

Tropics: **Regime of *Ro* $\gtrsim$ 1. Rotation of modest importance; lateral temperature differences tend to be small.**

Rotation causes east-west (zonal) banding in planetary atmospheres



Hayashi et al. (2000)

Even Venus (rotation period 243 Earth days) is banded, suggesting that transition to a banded flow can happen at *very low* rotation rates

Most planetary atmospheres should exhibit a banded flow pattern!